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ABSTRACT

This teacher's guide for a semester course in trigonometry is prepared for use with the text "Plane Trigonometry with Tables" by E. R. Heineman. Included is a daily schedule of topics for discussion and homework assignments. The scope of each lesson and teaching suggestions are provided. The content for the course includes trigonometric functions, solution of right triangles, trigonometric equations and identities, oblique triangles, and inverse trigonometric functions. Also included are two supplementary units on special right triangles and set theory. (Author/CT)

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BALTIMORE COUNTY PUBLIC SCHOOLS

A TENTATIVE GUIDE

TRIGONOMETRY

1965

CURRICULUM GUIDE

TRIGONOMETRY

A Tentative Guide

Prepared for Use with the Text

Plane Trigonometry with Tables

by

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Guide Produced by

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Towson, Maryland
1965

Foreword

This tentative guide has been written to aid teachers in presenting an updated course in trigonometry. The function concept as a set of ordered pairs and the analytic approach are key features of this guide. The lessons as well as the homework assignments are given as suggestions. The teacher should feel free to modify the lesson structure as may be appropriate for the student in his class. This guide serves as a framework within which the teacher should base the course in trigonometry. If teachers desire to alter the course to a considerable degree, they should discuss this with their department chairman or supervisor. Since this guide is tentative and will be revised after being used in the classroom, teachers are encouraged to offer suggestions and criticisms which will strengthen the outline.

Teachers should not overlook the excellent transparency projectuals which are in each mathematics department. Visual and graphic means should be used whenever possible.

MATERIALS OF THE COURSE IN TRIGONOMETRY

Plane Trigonometry with Tables by E. R. Heineman will be used as the basic text.

The references to the right of the scope in the outline indicate helpful or additional materials for the teacher and pupils.

Code		Title of Reference
RWM	-	Rosenbach, Whitman, Moskowitz. Plane Trigonometry with Tables. New York: Ginn and Company. 1943
FSM	-	Freilich, Shanholt, McCormack. Plane Trigonometry. Morristown, N. J.: Silver Burdett Company. 1957.
B	-	Brant, Vincent. Relations, Functions, and Graphs. Towson, Maryland: Board of Education of Baltimore County. 1962.
AK	-	Aiken and Beseman. Modern Mathematics: Topics and Problems. (Teacher's Edition). New York: McGraw-Hill Book Company.
BW	-	Butler and Wren. Trigonometry for Secondary Schools. Boston: D.C. Heath and Company. 1948.
M	-	May, K. O. Elements of Modern Mathematics. Reading, Massachusetts: Addison-Wesley Publishing Company. 1959.
Br	-	Brady Transparency Projectuals

CONTENTS

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE | PAGE
	UNIT ONE The Trigonometric Functions	
1	Introduction (1-1) Trigonometry is a subject which is important in physics and engineering as well as in other branches of mathematics. Two hundred years ago, it was a college subject. It was studied by sea captains for navigation, by surveyors to chart the colonial wilderness, and by ministers to calculate the date of Easter. However, with new navigational methods, specialization of surveying, and the assistance of observatories in calculating the dates of Easter, the practical measurement aspect of trigonometry has declined in importance. Today, the analytic treatment of trigonometric functions is receiving greater emphasis. This course will stress this modern point of view as well as the solution of triangles.	H.... 1
	Directed Segments (1-2)	H.... 2
	The Rectangular Coordinate System (1-3) Coordinate axes : x-axis, y-axis Origin Plotting a point Radius vector Quadrants	H.... 2, 3
	The Distance Formula (1-4)	H.... 4, 5
	SUGGESTED ASSIGNMENT H....pp. 5, 6 : Ex. 1, 5, 7, 9, 10, 11, 13	

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE | PAGE
2	Trigonometric Angles (1-5) Students who have studied the SMSG Geometry with Coordinates have been taught that an angle is the union of two noncollinear rays with a common endpoint. Such a concept excludes the zero angle, and angles equal to or greater than 180°. Students who have taken the course in regular Geometry have been taught that an angle is the union of two distinct rays which have a common endpoint. Such a concept excludes the zero angle and angles greater than 180° but includes an angle of 180°. It should be pointed out to the student that the restrictions of degree measure as noted above were necessary for a postulational treatment of angles. The student should be taught that certain extensions and modifications of the angle concept will be made in this course since the purposes in trigonometry are different from synthetic geometry. For example, engineers who deal with things like rotating shafts of motors will find such angles as 0°, 180°, 270°, 1750°, -50°, and others quite useful in his work. Thus trigonometry will deal with measures of angles which were not permitted in synthetic geometry on the basis of the postulates. The dynamic interpretation of an angle as the rotation of a terminal ray with respect to an initial ray is also useful in trigonometry. Experience has shown that students become quite confused with the angle concept unless teachers carefully explain the distinction in the purposes of the course.	H.... 5

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
2 (cont'd)	The text offers a definition of "angle" in terms of "an amount of rotation." The text uses the idea of angle in two senses: one, as the name of the angle, and the other, as the measure of the angle. For example, the symbol " θ " is used in some cases as the name of the angle, and in other cases as the number of degrees or radians as the measure of the angle. It would be more precise if the symbol "θ" were used as the name of the angle, and the symbol "$m(\theta)$" were used as the real number which is the measure of the angle. However, the student will have to judge in what sense the symbol "θ" is used in this text. It is well for the teacher to use the following ideas in clarifying the dual use of "angle." The trigonometric angle is the union of two rays, (not necessarily different) having a common endpoint. The measure of a trigonometric angle is the number assigned as the magnitude of the angle in terms of sexigesimal or radian measure. Observe that this definition permits the measure of angles 0, 180, as well as those which are greater than 180 or negative. The angle is still the union of two rays, not necessarily different. It is still a set concept. The measure of an angle is different than the angle. The measure is a number; the angle is a set of points. Make an agreement with the class that the symbol "θ" as used in the text may have two meanings -- one as a set of points and the other as a number. The appropriate use may be judged from the context of the discussion. It has been shown that students accept this dual use of "θ" without much difficulty provided the above ideas are explained.		

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	PAGE
2 (cont'd)	a. Initial ray b. Terminal ray c. Vertex d. Positive angle Review the following: The notation "$x > 0$" for real numbers means "the real number x is greater than zero." Emphasize the translation of "$x > 0$" as <u>x is positive.</u> In similar fashion, stress the translation of "$\theta > 0$" as "θ is positive" Actually, it is the number of degrees or radians which is positive. This is another example of the dual use of θ mentioned before. e. Negative angle Stress the translation of "$\theta < 0$" as "θ is negative" SUGGESTED ASSIGNMENT H .... p. 7 : Ex. 1 - 17	H.... RWM	7 10-12

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE | PAGE
3	Special Right Triangles The special right triangles such as the a. 3, 4, 5 triangle b. 5, 12, 13 triangle c. 30-60-90 triangle d. 45-45-90 triangle are used extensively in this course. The teacher should review this topic carefully. Provide sufficient practice in the 30-60-90 and 45-45-90 triangles so that the students may be able to calculate mentally the lengths of the two remaining sides of the triangle when they are given the length of any <u>one</u> side. This skill will be most useful in finding the values of the trigonometric functions of special angles. Use the unit in Appendix I on Special Right Triangles in the Geometry Course of Study. Copies of this unit may be obtained from your department chairman and <u>loaned</u> to students for this lesson. This is essentially review work so that the teacher can easily provide refresher experiences. SUGGESTED ASSIGNMENT Appendix I : pp. 125 - 127 : Ex. 1 - 10	Geometry Course of Study Appendix I Special Right Triangles

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE | PAGE
4	Radian Measure Up to this time students have been familiar with the measure of angles based upon the sexagesimal system. It is now most important that students be introduced to the concept of radian measure. You will note that the text defers this topic until later. However, radian measure and its use throughout the course is so important that it should be presented at this time. From this point on, the teacher should use radian measure and sexagesimal measure interchangeably so that the student becomes proficient with both measures. The teacher should stress that the radian is a number---not an angle. A radian is a special number which measures a central angle whose arc has a length equal to the radius of the circle. The number of radians in a central angle tells how many times the radius is "wrapped around" the arc of the central angle. The teacher should use a "radian wheel" device with string to illustrate that the sexagesimal measure of the central angle subtended by a <u>chord</u>, whose length equals the radius, is 60; in contrast, the sexagesimal measure of the central angle whose <u>arc</u> length equals the radius, is less than 60.	H.... 65-69 RWM 83-88 SMSG, Elementary Functions.. 243-245

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
4 (cont'd)	The teacher should develop the relationship between radian and sexagesimal measure in an intuitive fashion. It is helpful to write out the words "radian" and "degrees" in using the approach suggested below.		

Since the radius of a circle is contained 2π times in the circumference of a circle, we may say:

$$2\pi \ \underline{\text{radians}} = 360 \ \underline{\text{degrees}}$$

$$\pi \ \underline{\text{radians}} = 180 \ \underline{\text{degrees}} \quad \left(\text{multiplying by } \frac{1}{2}\right)$$

From this last basic formula the students can always obtain the relationships:

$$1 \ \underline{\text{radian}} = \frac{180}{\pi} \text{ degrees} \quad \left(\text{multiplying by } \frac{1}{\pi}\right)$$

or

$$\frac{\pi}{180} \text{ radians} = 1 \ \underline{\text{degree}} \quad \left(\text{multiplying by } \frac{1}{180}\right)$$

SUGGESTED ASSIGNMENT

H....pp. 68, 69 : Ex. 1, 2, 3, 5, 6, 7, 9, 10, 11, 17, 19, 21, 23, 25, 27, 29, 57, 58, 59

<table>
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<tr><td>5</td><td>Length of a Circular Arc

Note: Linear and Angular Velocity may be presented later in the course, if time permits.

SUGGESTED ASSIGNMENT

H....pp. 71-73 : Ex. 1, 2, 5, 6, 7, 13, 14, 18, 19</td><td>H....
RWM</td><td>69-73
88-90</td></tr>
<tr><td>6</td><td>Relations, Function, and Graphs

The text mentions the idea of a trigonometric function as a special set of ordered pairs, but for the most part treats trigonometric functions as special ratios. In this course, however, we shall emphasize a more modern approach. Hence, it will be necessary to review concepts of sets, relations, functions, and graphs which students have studied in algebra and geometry. This should be a rapid review. Student review sheets of elementary concepts should be obtained from your department chairman.

In this lesson, the teacher should review the following ideas:

a. Set is an undefined term. We may describe a set as a collection of things or objects.

b. Set notation

1. The <u>phrase</u> method

Example : the set of the (names of) the vertices of △ ABC.</td><td>B....

AK...</td><td>Relations, Functions, and Graphs

1-139</td></tr>
</table>

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
6 (cont'd)	2. The roster (listing) method Example . Using same set as in (1) ; $\{ A, B, C \}$ 3. The rule method Example : Using same set as in (1) and (2) ; $\{ x \mid x \text{ is a vertex of } \triangle ABC \}$ c. The empty set The set which contains no elements is the empty set. The symbol for the empty set is "ϕ" d. Subset of a given set Set A is a subset of set B (denoted "$A \subset B$") if and only if each element of A is also an element of B. Example: Let $B = \{1, 2, 3, 4, 5\}$ $A = \{1, 3, 4\}$ $C = \{4\}$ $D = \{2, 5, 3, 4, 1\}$ Then : $A \subset B$; $C \subset B$; $D \subset B$; $C \subset A$; Also $\phi \subset A$; $\phi \subset B$; $\phi \subset C$; $\phi \subset D$ e. Cartesian Product The Cartesian Product of two sets A and B (not necessarily different) is the set of all ordered pairs in which the first coordinate belongs to A and the second coordinate belongs to B. The Cartesian Product of A and B is denoted as $A \times B$ (read "A cross B") Example: Let $A = \{ a, b, c, \}$; Let $B = \{ 1, 2 \}$ Then, $A \times B = \{(a, 1), (b, 1), (c, 1), (a, 2), (b, 2), (c, 2)\}$		

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE | PAGE
6 (cont'd)		

f. Relation

A relation is a set of ordered pairs.

or

A relation from A to B is a subset of $A \times B$.

Example: Let $A = \{a, b, c\}$; let $B = \{1, 2\}$

Some relations from A to B are:

Relation $R = \{ (a,1), (b,2), (c,1) \}$
Relation $Q = \{ (a,2) \}$
Relation $T = \{ (a,1), (b,1), (c,1) \}$

Of course, $A \times B$ is also a relation.
Furthermore, the empty set, ϕ, is a relation.

g. Domain and range of a relation

The domain of a relation is the set of all the first coordinates of the ordered pairs of the relation.

The range of a relation is the set of all the second coordinates of the ordered pairs of the relation.

Example: Let Relation $T = \{ (a,2), (b,3), (a,5), (c,2) \}$

Then, Domain of $T = \{a, b, c\}$
Range of $T = \{2, 3, 5\}$

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
6 (cont'd)	h. Function		

h. Function

A function from A to B is a special kind of relation such that for each first coordinate there is one and only one second coordinate.

Stated differently, no two ordered pairs of a function may have the same first coordinate. Hence, the first coordinates in the ordered pairs of the function must be different. However, the second coordinate may be the same.

Examples: $D = \{ (1, a), (2, b), (3, c) \}$ is a function.
$H = \{ (1, a), (2, b), (1, c) \}$ is not a function; it is a relation.
$K = \{ (1, a), (2, a), (3, a) \}$ is a function---- a constant function.

Note: Domain of $K = \{ 1, 2, 3, \}$; Range of $K = \{a\}$

$I = \{ (x, y) \mid y = x \}$

Note: Domain of I = the set of real numbers
Range of I = the set of real numbers

i. Tests for a function

1. The ordered pair test

A relation is a function prpvided the first coordinates of the ordered pairs are different.

2. The vertical line test

The relation is a function provided every vertical line intersects the graph of the relation in exactly one point. If at least one vertical line intersects the graph in more than one point, then the graph does not represent a function, but a relation.

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
6 (cont'd)	j. Value of a function The teacher should be very careful and precise with the language used in discussing functions. In the past, such language has been used loosely and imprecisely, leading to fuzzy and vague notions. This is especially true when discussing the idea of "the value of the function." The following ideas should be emphasized when discussing a function such as: $f = \{ (x, y) \mid y = 5x \}$ 1. The function f is the entire set of ordered pairs 2. The function is <u>NOT</u> "y = 5x" Instead, "y = 5x" is a sentence which defines the function. This sentence assigns to each real number used as a first coordinate another real number 5 times as large for its corresponding second coordinate. 3. "y", "5" or "f(x)" are different names for the second coordinate. Thus, the function f might also be written in the following notations: $f = \{ (x, y) \mid y = 5x \}$ $f = \{ (x, f(x)) \mid f(x) = 5x \}$ $f = \{ (x, 5x) \mid x \text{ is a real number} \}$ 4. "f(x)" is translated as "f at x" or "f of x"		

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
6 (cont'd)	5. In describing a function, such as f above, the teacher should be careful to use the following language: "the function f defined by $y = x$", Do <u>not</u> say: "the function $y = x$" The teacher should capitalize on every opportunity to use these ideas throughout the course. This will result in extra dividends when the unit on graphing the trigonometric functions is taught. SUGGESTED ASSIGNMENT (The teacher should ditto copies of this assignment or obtain mimeographed copies from the department chairman) <u>Written Assignment</u> 1. Construct the cartesian produce of $A \times B$ where $A = \{ r, t, k \}$ and $B = \{ 6, 16, 1526 \}$ 2. Construct the cartesian product of $R \times S$ where $R = \{ 0, 2, 4, 6 \}$ and $S = \{ 1, 3, 5 \}$ 3. Construct the cartesian product of $E \times D$ where $D = \{ a \}$ and $E = \{ 1 \}$ Each of the following relations is a relation from R to R where R is the set of real numbers. In each example, complete the following parts: a. Draw the graph of the relation. b. State whether the relation is a function. c. State the domain of the relation. d. State the range of the relation.		

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
6 (cont'd)	SUGGESTED ASSIGNMENT (continued) 4. $R = \{ (x, y) \mid y = x \}$ 5. $S = \{ (x, S(x)) \mid S(x) = \frac{1}{2} x \}$ 6. $T = \{ (x, y) \mid y > 5x \}$ 7. $U = \{ (x, y) \mid y = x^2 \}$ 8. $V = \{ (x, V(x)) \mid V(x) = 2x - 5 \}$ 9. $W = \{ (x, y) \mid y = x^3 \}$ 10. $X = \{ (x, y) \mid x^2 + y^2 = 25 \}$ 11. $Y = \{ (x, y) \mid xy = 12 \}$ 12. $G = \{ (x, G(x)) \mid G(x) = 5 \}$ 13. $H = \{ (x, y) \mid x = 3 \}$ Study Assignment: Study Mimeographed Sheet, "Review of Elementary Set Concepts"		

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE | PAGE
7	Definitions of the Trigonometric Functions of a General Angle (1-7)	H....7-10

The teacher should present the definitions of the trigonometric functions in a manner consistent with the concept of function as reviewed in the previous lesson. Observe that the text does mention the function as a special set of ordered pairs, but then reverts to loose expressions when speaking of the function and its value. The teacher should define the trigonometric functions as follows:

$$\text{sine (or sin)} = \{(\theta, \sin\theta) \mid \sin\theta = \frac{y}{r}\}$$

Observe that

(1) The symbol "sin" indicates a set of ordered pairs

(2) θ is a number---a number that measures the magnitude of the angle in degrees or radians.

(3) Sin θ is a number ---- the number which is assigned as the second coordinate by the defining sentence

$$\sin\theta = \frac{y}{r}$$

(4) Sin θ is <u>NOT</u> the function.

Sin θ is the name of the second coordinate.

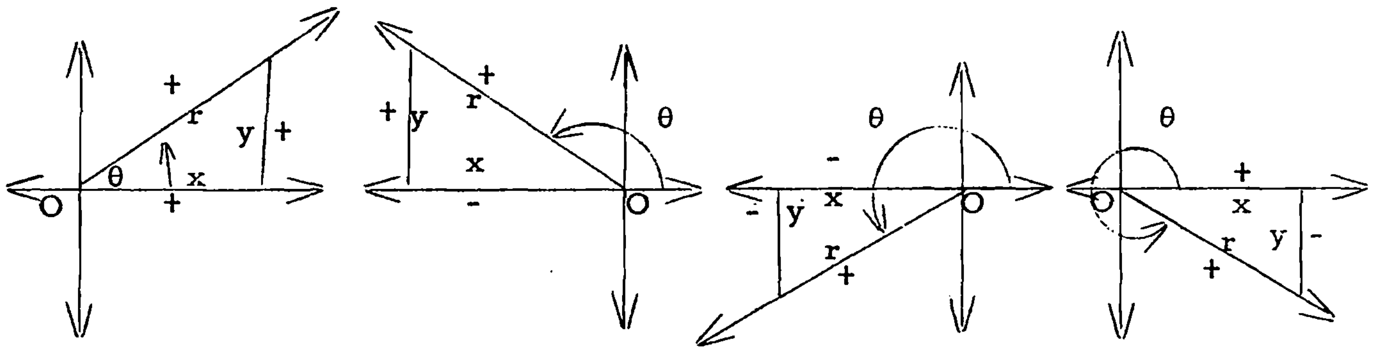
Sin θ is the <u>VALUE</u> of the function at θ.

(5) Each ordered pair of the sine function has real numbers for the first and second coordinates.

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
7 (cont'd)	cosine (or cos) = $\{(\theta, \cos\theta) \mid \cos\theta = \frac{x}{r}\}$ tangent (or tan) = $\{(\theta, \tan\theta) \mid \tan\theta = \frac{y}{x}\}$ cotangent (or cot) = $\{(\theta, \cot\theta) \mid \cot\theta = \frac{x}{y}\}$ secant (or sec) = $\{(\theta, \sec\theta) \mid \sec\theta = \frac{r}{x}\}$ cosecant (or scs) = $\{(\theta, \mathrm{scs}\,\theta) \mid \csc\theta = \frac{r}{y}\}$		

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE | PAGE
7 (cont'd)	Thus, the trigonometric functions are called sin, cos, tan, cot, sec, and csc. The values of the trigonometric functions (or second coordinates of the ordered pairs) are sin θ, cos θ, tan θ, cot θ, sec θ, and csc θ. Emphasize the fact that the domain of each of the above functions consists of all real numbers θ for which the corresponding ratios have a nonzero denominator, since division by zero is impossible (not defined). Stress the fact that the value of the trigonometric functions depends solely upon the magnitude of the angle and not upon the lengths of the sides used in the ratios.	
	Consequences of the Definitions (1-8) 1. Reciprocal relations 2. Theorem on coterminal angles: The value of the trigonometric function of an angle is equal to the value of the trigonometric function of any angle coterminal the given angle.	H....10, 11
	SUGGESTED ASSIGNMENT H....p. 14; Ex. 1, 2, 6, 7, 19, 21, 22, 23, 25, 26, 27, 29, 30	

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE | PAGE
8 (cont'd)	Signs of the values of the trigonometric functions The presentation of this topic in the text is poor and should not be used. Instead, the teacher should provide an opportunity for discovery in this topic by presenting the following diagrams and charts. Have the students complete the chart so that they discover the generalization and formulate the rule.	H....11 RWM 17,18



Quadrant / Function value	I	II	III	IV
sin θ	$\frac{+}{+} = +$	$\frac{+}{+} = +$		
cos θ	$\frac{+}{+} = +$	$\frac{-}{+} = -$		
tan θ	$\frac{+}{+} = +$			
cot θ	$\frac{+}{+} = +$			
sec θ	$\frac{+}{+} = +$			
csc θ	$\frac{+}{+} = +$			

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE | PAGE
8 (cont'd)	SUMMARY CHART FOR SIGNS OF FUNCTION VALUES	

$\sin\theta > 0$ $\csc\theta > 0$ Q II	all function values are positive Q I
Q III $\tan\theta > 0$ $\cot\theta > 0$	Q IV $\cos\theta > 0$ $\sec\theta > 0$

Use of inequality symbols

1. $\sin\theta > 0$ means that the value of the sine is positive.
2. $\sin\theta < 0$ means that the value of the sine is negative.

Observe the incorrect language used by the text in Examples 31-38 on page 15. The interpretation of "$\sin\theta = +$" means that a number is the same as the positive symbol. This is incorrect, because equality in a sentence means "names the same object as", or "is another name for." Direct your students to rewrite these in proper form; example 31 should be properly written as

$$\sin\theta < 0 \text{ and } \tan > 0.$$

The use of set intersection is helpful in solving problems such as example 31 above. The presentation of the solution might be as follows:

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE | PAGE
8 (cont'd)		

Let x be the quadrant or quadrants in which the given angle terminates.

Now, $M = \{ x \mid \sin\theta < 0 \} = \{ III, IV \}$

$N = \{ x \mid \tan\theta > 0 \} = \{ I, III \}$

Hence, $M \cap N = \{ x \mid \sin\theta < 0 \} \cap \{ x \mid \tan\theta > 0 \}$

$= \{ III, IV \} \cap \{ I, III \}$

$= \{ III \}$

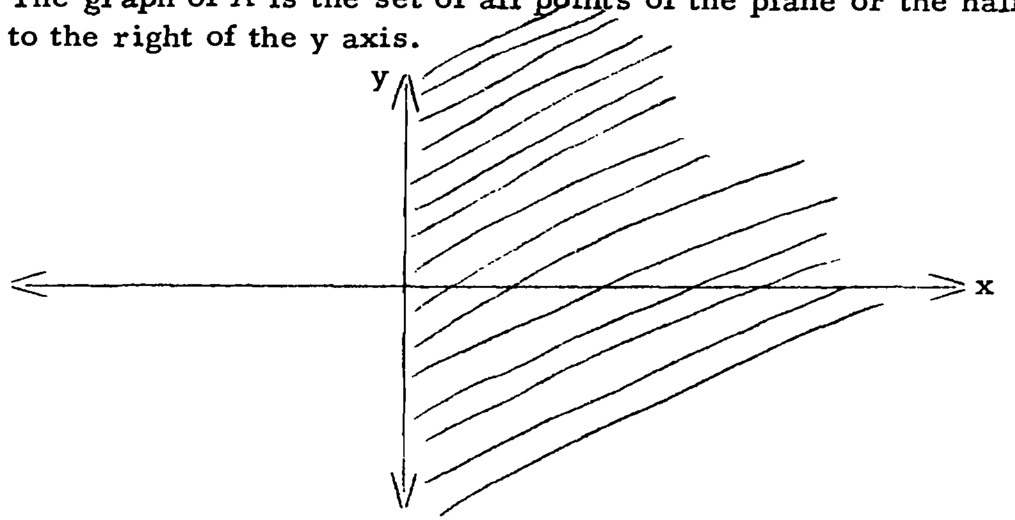
Representation of the quadrants by set notation

Let R be the set of real numbers.

Then $R \times R$ is the set of ordered pairs of real numbers, and its graph is the entire plane.

Hence, $A = \{ (x,y) \mid x > 0 \}$;

The graph of A is the set of all points of the plane or the half-plane to the right of the y axis.

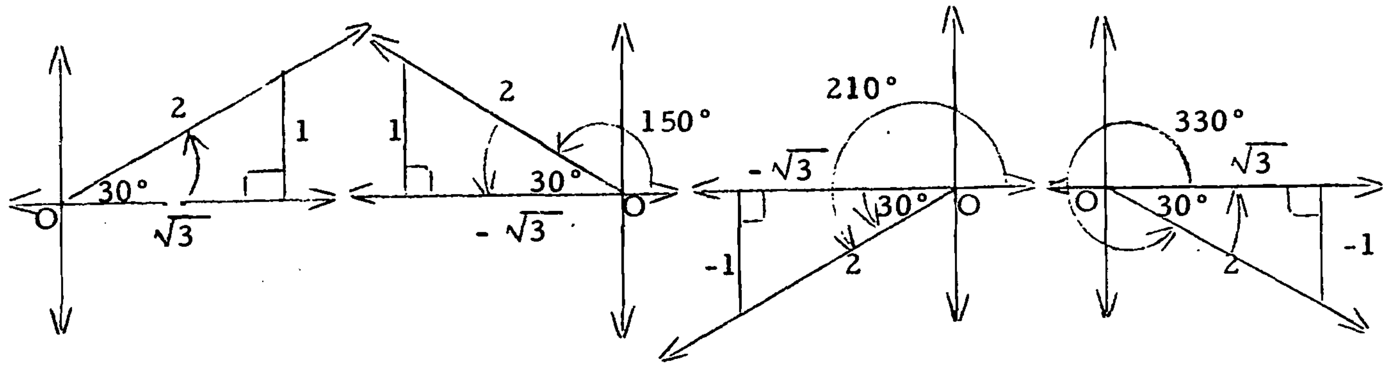


LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
8 (cont	$B = \{ (x, y) \mid x < 0 \}$ The graph of B is the set of all points of the plane or the half-plane to the left of the y axis. $C = \{ (x, y) \mid y > 0 \}$ The graph of C is the set of all points of the plane or the half-plane above the x axis. $D = \{ (x, y) \mid y < 0 \}$ The graph of D is the set of all points of the plane or the half-plane below the x axis. Hence, $A \cap C = \{ (x, y) \mid x > 0 \text{ and } y > 0 \}$; its graph is Quadrant I. $B \cap C = \{ (x, y) \mid x < 0 \text{ and } y > 0 \}$; its graph is Quadrant II. $B \cap D = \{ (x, y) \mid x < 0 \text{ and } y < 0 \}$; its graph is Quadrant III. $A \cap D = \{ (x, y) \mid x > 0 \text{ and } y < 0 \}$; its graph is Quadrant IV.		

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
8 (cont'd)	HOMEWORK ASSIGNMENT The teacher should either ditto or write on the board the problems below which are not in the text. 1. In what quadrants must the angle whose measure is θ terminate under the following conditions: (a) $\sin\theta > 0$; (b) $\cos\theta < 0$; (c) $\tan\theta < 0$; (d) $\sec\theta > 0$ 2. (Refer to page 11 in the text for model illustrations) Express the function values of sin, cos, tan of the angles given below as function values of angles between zero and 360. (a) $390°$ (b) $890°$ (c) $-34°$ (d) $-590°$ In problems 31 - 38 on page 15, (1) Rewrite the problem in correct notation using inequalities. (2) Identify the quadrant in which the given angle, whose measure is θ, terminates in order to satisfy the conditions.		

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
9	Function values of special angles It is wise at this time to provide further practice in finding the function values of angles by using the ratios of the sides of the 30-60-90 and 45-45-90 triangles. The text does this in different sections. In this guide, however, the trigonometric function values of such angles as 330°, - 135°, 600° and the quadrantal angles are treated at one time. In this section teachers should use radian and sexagesimal measure interchangeably. The following ideas are important in this lesson: 1. The related (or reference angle) as the positive acute angle between the horizontal (or x) axis and the terminal side of the given angle. 2. The related (or reference) triangle as the triangle formed by drawing segments representing the abscissa and the ordinate of a point on the terminal side. 3. The 30-60-90 triangle as a special related triangle. 4. The 45-45-90 triangle as a special related triangle. 5. The unit circle and the coordinates of the points where the x and y axes intersect the circle. The teacher should present the diagrams and charts on the next page and let the students complete the chart after a suitable explanation has been made concerning the procedure in finding the function values.	H. . . . RWM	21-23 50-52 21-26

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	PAGE
9 (cont'd)	A. The function values of angles related to an angle of $\frac{\pi}{6}$ radians or 30°.		

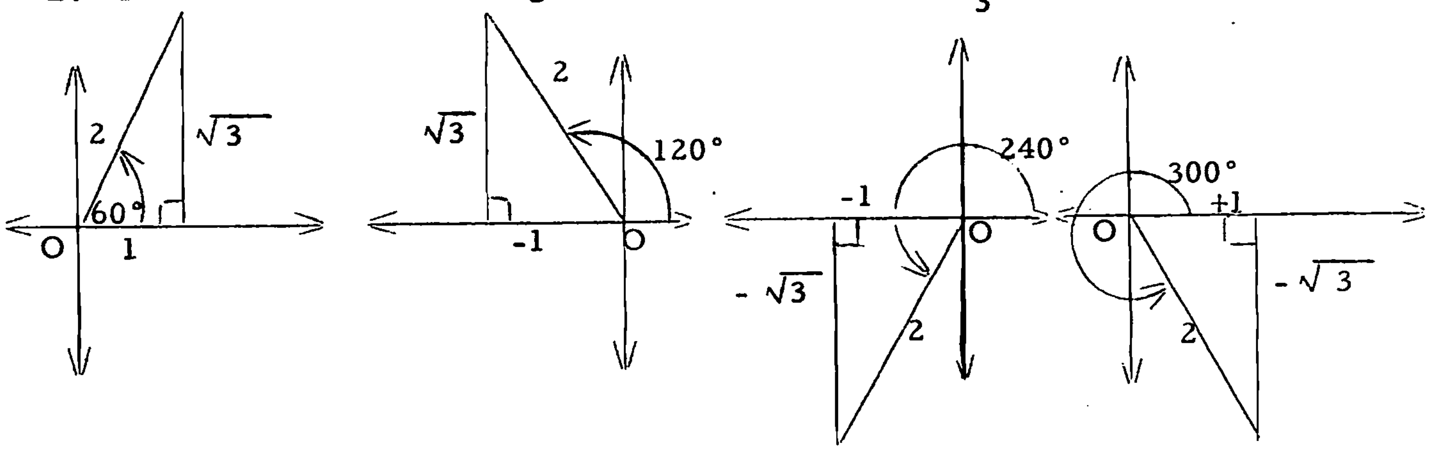


COMPLETE THE CHART

Function \ Angle	$\frac{1}{6}\pi$ or 30°	$\frac{5}{6}\pi$ or 150°	$\frac{7}{6}\pi$ or 210°	$\frac{11}{6}\pi$ or 330°
$\sin\theta$	$\frac{1}{2}$			
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{2}$		
$\tan\theta$	$\frac{\sqrt{3}}{3}$			
$\cot\theta$		$-\sqrt{3}$		
$\sec\theta$				
$\csc\theta$				

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE | PAGE
9 (cont'd)		

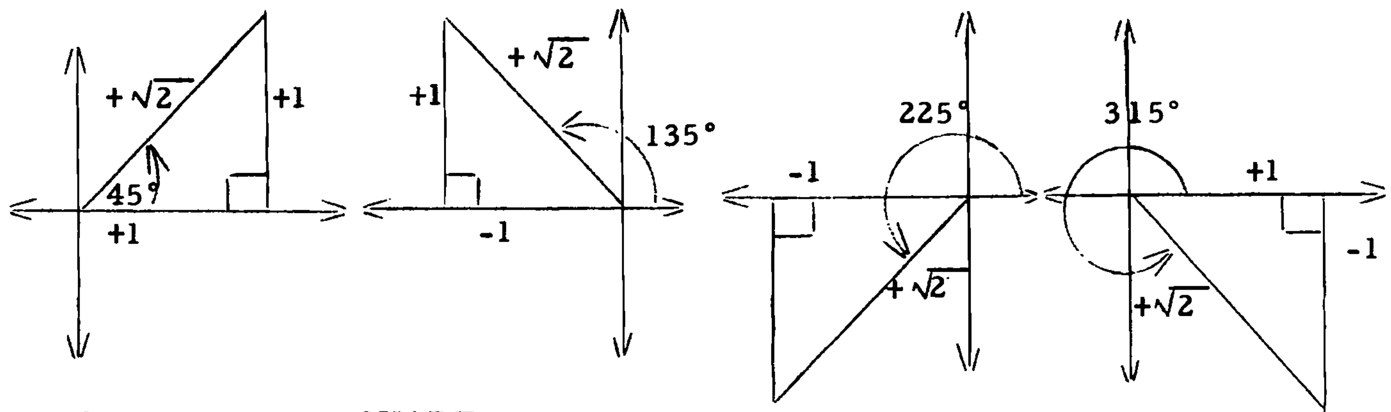
B. The function values of angles related to an angle of $\frac{\pi}{3}$ radians or 60°.



COMPLETE THE CHART

Angle / Function	$\frac{1}{3}\pi$ or 60°	$\frac{2}{3}\pi$ or 120°	$\frac{4}{3}\pi$ or 240°	$\frac{5}{3}\pi$ or 300°
sin θ	$\frac{\sqrt{3}}{2}$			
cos θ	$\frac{1}{2}$	$-\frac{1}{2}$		
tan θ	$\sqrt{3}$			
cot θ		$\frac{-\sqrt{3}}{3}$		
sec θ				
csc θ				

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE | PAGE
9 (cont'd)	C. The function values of angles related to angle of $\frac{\pi}{4}$ radians or 45°.	

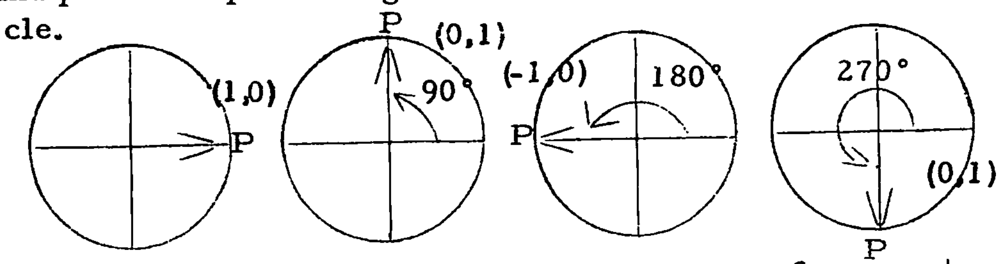


COMPLETE THE CHART

Angle / Function	$\frac{1}{4}\pi$ or 45°	$\frac{3}{4}\pi$ or 135°	$\frac{5}{4}\pi$ or 225°	$\frac{7}{4}\pi$ or 315°
$\sin\theta$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$			
$\cos\theta$		$-\frac{\sqrt{2}}{2}$		
$\tan\theta$	1			
$\cot\theta$			1	
$\sec\theta$	$\sqrt{2}$			
$\csc\theta$				

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE | PAGE
9 (cont'd)	D. Quadrantal angles; that is, angles whose measure is 0, $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$ radians	

Use a unit circle and point P representing the intersection of the radius vector and the circle.



	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
Abscissa (x)	1	0	-1	0
Ordinate (y)	0	1	0	-1
Radius vector (r)	1	1	1	1

Angle / Function	0 or 0°	$\frac{1}{2}\pi$ or 90°	π or 180°	$\frac{3}{2}\pi$ or 270°
$\sin\theta$	$\frac{y}{r}=\frac{0}{1}=0$		$-\frac{1}{1}=-1$	
$\cos\theta$	$\frac{x}{r}=\frac{1}{1}=1$			
$\tan\theta$	$\frac{y}{x}=\frac{0}{1}=0$	$\frac{1}{0}=$ not defined		
$\cot\theta$	$\frac{x}{y}=\frac{1}{0}=$ not defined			
$\sec\theta$				
$\csc\theta$				

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
9 (cont'd)	The teacher may wish to let the students complete the charts for homework and announce a short quiz on this topic for the next day, or the teacher may wish to have the charts completed in class and assign additional problems for home study. In that case, the suggested assignment is given below. SUGGESTED ASSIGNMENT H....p.23: Ex. 1, 2, 5, 6 H....p.61,62: Ex. 1, 2, 6, 9, 10 H....p.69: Ex. 37, 39, 41, 43, 45, 49, 51 97		

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE | PAGE
10	**Given One Trigonometric Function Value of an Angle, To Draw the Angle and Find the Other Trigonometric Functions.**	
	Students should be shown that this topic actually reverses the procedure practiced in the previous lessons. In prior sections, students were primarily interested in finding the <u>second</u> coordinate of an ordered pair when the first coordinate was given; that is,	
	To find sin 30° means to fill in the slot in the ordered pair (30°, |______|)	
	In this new section, students are concerned with finding the <u>first</u> coordinate of an ordered pair when the second coordinate is given; for example:	
	To find θ when $\sin\theta = \frac{1}{2}$ means to fill in the slot in (|______|, $\frac{1}{2}$)	
	The teacher should take care not to call this the inverse function process since the inverse of sin would require (1) the interchange of the first and second coordinates (2) the resulting set of ordered pairs also be a function	M. . . .284-294
	Note that ordered pairs in such an interchange would give rise to such ordered pairs as	
	$(\frac{1}{2}, 30°)$, $(\frac{1}{2}, 150°)$, $(\frac{1}{2}, 390°)$, $(\frac{1}{2}, 510°)$, etc.	
	Since the first coordinates in the ordered pair of a function must be different, interchanging the coordinates would not result in a function. Of course, with suitable restrictions, a function may result.	
	The following procedure is helpful in the solution of a problem such as	
	Given: $\sin\theta = \frac{1}{2}$; locate the terminal side of the angle.	

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE | PAGE
10 (cont'd)		

Procedure for solution of: Given $\sin\theta = \frac{1}{2}$; draw the angle in standard position.

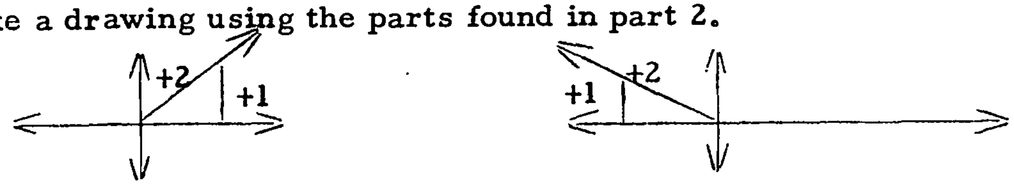
1. Determine the quadrant or quadrants from the sign of the function value. Keep emphasizing the importance of this first step. In the above problem the sine value is positive and hence the angle must lie in Quadrant I or Quadrant II.

2. Write the defining sentence of the sin function, and the given ratio as below. This will enable the student to assign lengths to a related (or reference) triangle.

$$\sin\theta = \frac{y}{r} = \frac{+1}{+2}\text{; Hence } y = +1\text{ ; } r = +2$$

Of course, the fraction cannot be represented as $\frac{-1}{-2}$ because the radius vector is defined to be a positive quantity. However, in the tangent or cotangent ratio, either the numerator or denominator can take on either positive or negative values. In the secant and cosecant ratio, the numerator must always be a positive quantity.

3. Make a drawing using the parts found in part 2.



4. The third side may be found by the Pythagorean relation. In this case, students should recognize the ratio of the sides $1 : \sqrt{3} : 2$ as belonging to the 30-60-90 triangle.

5. Hence the related angle is 30°, and θ = 30° and 150°.

SUGGESTED ASSIGNMENT

H....p.17; Ex. 1, 3, 5, 7, 9, 11

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
11	Reinforcement of Lesson 10 SUGGESTED ASSIGNMENT The exercises in RWM are excellent and more challenging. You will want to copy these on the board or ditto them. RWM.... pp. 20,21: Ex. 1, 3, 17, 19, 21, 24(a)		
12	Review for test		
13	Test		

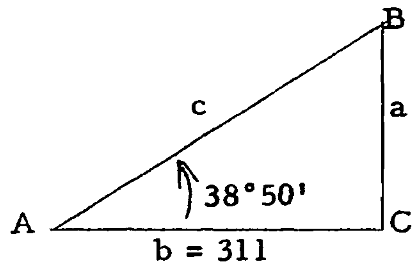
LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	PAGE
	UNIT TWO <u>Solution of Right Angles</u>		
14	Trigonometric Functions of an Acute Angle (2-10)	H.... RWM	18, 19 28-30
	Cofunctions (2-11) Theorem 1: Any trigonometric function value of an acute angle is equal to the cofunction value of its complementary angle.	H.... RWM	20 30
	Theorem 2: If two acute angles, A and B, have any function value of one equal to the confunction value of the other, then the angles A and B are complementary. (Converse of Theorem 1)	RWM	30
	The topic of variation of the function values at this point is too limited in scope. Hence, it will be deferred to a later section dealing with line representations of trigonometric functions.		
	SUGGESTED ASSIGNMENT H....p.19: Exercise 5: Examples 1, 2, 3, 4 H....p.23: Exercise 6: Examples 9, 10, 11, 13, 14 RWM...p.31: Examples 1, 3, 5, 12, 17		

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	PAGE
15	Tables of Trigonometric Function Values (2-14)	H....	23, 24
	Given an Angle, To Find the Value of One of Its Functions (2-15)	H....	24
	Given the Function Value of an Angle, To Find the Angle (2-16)	H....	24
	SUGGESTED ASSIGNMENT		
	H....p.25: Exercise 7; Examples 1, 2, 3, 5, 6, 7, 9, 10, 11, 13, 14, 15, 17, 18, 19, 21, 22, 23		
16	Interpolation (2-17)	H....	25-27
	SUGGESTED ASSIGNMENT		
	H....p.27, 28: Exercise 8: Examples 1, 2, 3, 9, 10, 11, 13 17, 18, 19, 21, 22		
17	Interpolation (Second day for further reinforcement of skills)	H....	25-27
	Approximations and Significant Figures (2-18) An excellent treatment of this topic may be found in Butler and Wren	H.... BW..	28-30 41-44
	SUGGESTED ASSIGNMENT		
	H....p.27, 28: Exercise 8: Examples 5, 6, 7, 14, 15 25, 26, 27, 29, 30		
	H....p.30: Exercise 9: Examples 1, 2, 3, 5, 6, 7, 9, 10, 11		

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE | PAGE
18	The Solution of Right Triangles (2-19)	H....30-33

Insist upon a neat format for the solution of the right triangles. Such a format might be patterned after the following:

Given: $\angle A = 38°50'$, $b = 311$



To find: $\angle B =$, $a =$, $c =$

(1) **To find $\angle B$**

$\angle B = 90° - 38°50'$

$\angle B = 51°10'$

(2) **To find a**

$\text{Tan } A = \frac{a}{b}$

$a = b \text{ Tan } A$

$a = 311 \text{ Tan } 38°50'$

$a = 311\ (0.8050)$

$a = 250$

(3) **To find c**

$\text{Cos } A = \frac{b}{c}$

$c = \frac{b}{\text{Cos } A}$

$c = \frac{311}{0.7790}$

$c = 399$

SUGGESTED ASSIGNMENT

H....pp. 33-34: Exercise 10: Examples 1, 3, 5, 7, 9, 15

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	PAGE
19	Angles of Elevation and Depression (2-20)	H....	34-36
	Bearing of a Line (2-20)	H....	34-36
	SUGGESTED ASSIGNMENT		
	H....pp. 36, 37: Exercise 11 : Examples 1, 3, 5, 7, 11, 13		
20	Angles of Elevation and Depression; Bearing of a Line (Second Day)		
	SUGGESTED ASSIGNMENT		
	H....pp. 36-39: Exercise 11: Examples 14, 15, 21, 22, 23, 28		
21	Review		
22	Test		

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE | PAGE

UNIT THREE

Line Representation of Trigonometric Function Values

Introduction

Previously, the trigonometric functions of the general angle have been expressed as the ratio of two segments represented by the abscissa, ordinate or the length of the radius vector of a point on the terminal side of the radius vector. It is possible and often convenient to represent the trigonometric function values as the lengths of a single line segment. This can be done by representing the angle in a unit circle, and then representing the trigonometric ratios with denominator 1.

It is unfortunate that the text which is excellent in many respects chooses to omit this material since it provides an interesting graphic method by which the student may chart the variation of the trigonometric function values. Furthermore, the line representations thus developed will be used to develop the basic trigonometric identities through graphic methods. The references by RWM and FSM provide an excellent presentation of this topic.

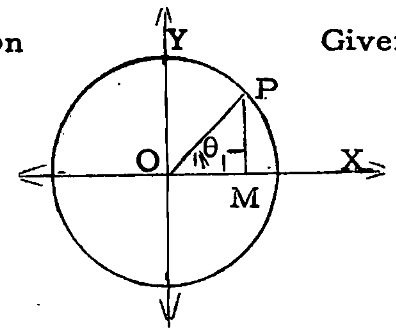
For better classes the teacher may wish to present the line representation of the sin values, and assign the line representation of the cos values for homework. Then the tan might be discussed in class with the cot assigned for homework; and the sec might be discussed in class with the csc assigned for homework.

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
	For slower classes, the teacher may wish to develop all of these in class. Some teachers prefer to teach tan and sec simultaneously and the cot and csc simultaneously. This is left to teacher preference and to his judgment of the best way to present it to the students in his class. In any case, the teacher should emphasize the following ideas: (1) domain and range of the function (2) amplitude and periodicity (3) graphical representation Ask your department chairman for the excellent transparency projectuals which you should use in presenting this topic.		

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE | PAGE
23	Line Representation of the Sin Function Value	H. . . . 76-82, 115-123 FSM. . .61, 63 66-68 RWM. 232-234 239-244

Line Representation of the Sin Function Value

A. Geometric derivation



Given: Unit circle O with angle θ, point P the intersection of the unit radius vector and the unit circle.

1. Draw $PM \perp OX$

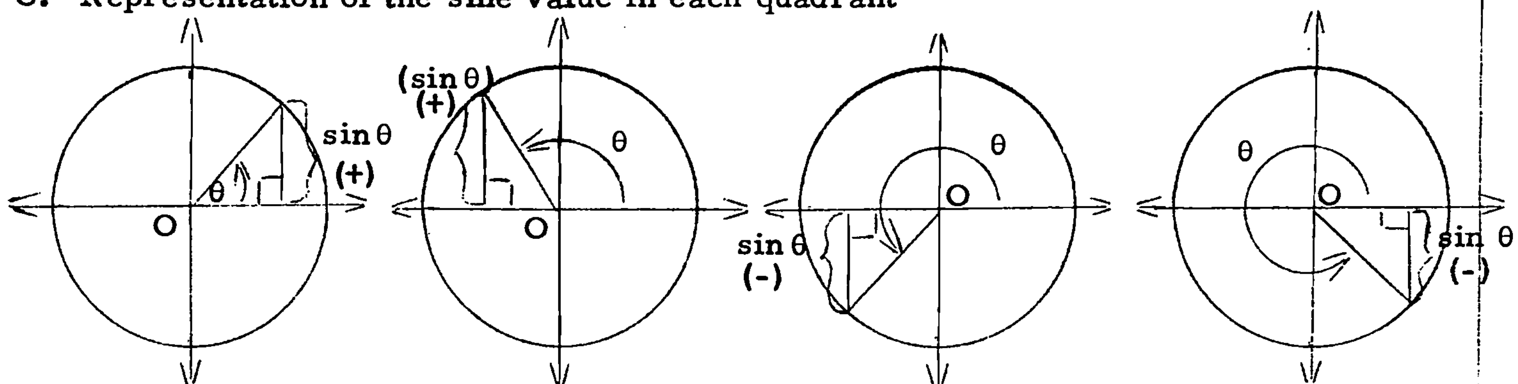
2. $\sin\theta = \frac{PM}{PO}$

$= \frac{PM}{1}$

$= PM$

B. Definition: The line representation of the sine is the ordinate of point P on the unit circle with radius vector OP. It is positive if measured above the x axis; negative, if measured below the x axis.

C. Representation of the sine value in each quadrant



LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
23 (cont'd)			

D. Complete the chart of variation for the sine values

Angle	0	$\frac{1}{6}\pi$	$\frac{1}{4}\pi$	$\frac{1}{3}\pi$	$\frac{1}{2}\pi$	$\frac{2}{3}\pi$	$\frac{3}{4}\pi$	$\frac{5}{6}\pi$	π	$\frac{7}{6}\pi$	$\frac{5}{4}\pi$	$\frac{4}{3}\pi$	$\frac{3}{2}\pi$	$\frac{5}{3}\pi$	$\frac{7}{4}\pi$	$\frac{11}{6}\pi$
Function value of sin	0	.5	.71													

E. Graphic representation of the sine function

1. Sine curve (sinusoid) defined by $y = \sin\theta$; domain; range
2. Period; amplitude; continuity
3. Graph of (a) the function defined by $y = \frac{1}{2}\sin\theta$
   (b) the function defined by $y = 2\sin\theta$
   (c) the function defined by $y = 3\sin\theta$

   Comparison of the graphs of (a), (b), (c) with respect to amplitude and period.
4. Graph of (a) the function defined by $y = \sin\frac{1}{2}\theta$
   (b) the function defined by $y = \sin 2\theta$
   (c) the function defined by $y = \sin 3\theta$

   Comparison of the graphs of (a), (b), (c), with respect to amplitude and period.
5. Generalizations concerning the period and amplitude

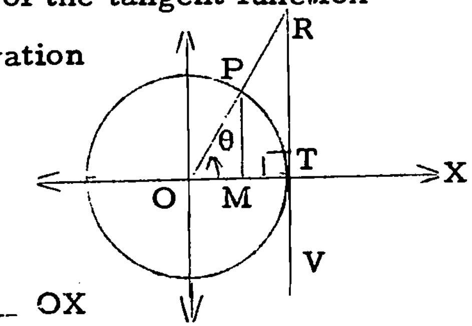
NOTE: To save time, the teacher may wish to let one row of students graph 3(a), another row 3(b), etc., and let the students draw the graphs on one set of coordinate axis to make comparisons and generalizations.

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	PAGE
23 (cont'd)	**SUGGESTED ASSIGNMENT** Repeat the classroom discussion for the line representation for cos values. Be sure to include the following ideas: A. Geometric derivation (Hint: Observe the abscissa of the point. B. Definition C. Line representation of the cos value in each quadrant D. Graphic representation of the cosine curve. 1. Cosine curve defined by $y = \cos\theta$; domain; range 2. Period; amplitude; continuity 3. Graph of (a) the function defined by $y = \frac{1}{2}\cos\theta$ (b) the function defined by $y = 2\cos\theta$ (c) the function defined by $y = 3\cos\theta$ Compare the graphs of (a), (b), (c) with respect to amplitude and period. 4. Graph of (a) the function defined by $y = \cos\frac{1}{2}\theta$ (b) the function defined by $y = \cos 2\theta$ (c) the function defined by $y = \cos 3\theta$ Compare the graphs of (a), (b), (c) with respect to period and amplitude. 5. Make a generalization concerning amplitude and period for the cos curves.	H.... FSM. RWM	76-78, 82, 115-123 61-63 66-68 232-234 239-245

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE | PAGE
24	IV. Line representation of the tangent function	H. . . . 82, 83 FSM. 61-63, 65 69-71 RWM 235-237 246

IV. Line representation of the tangent function

A. Geometric derivation

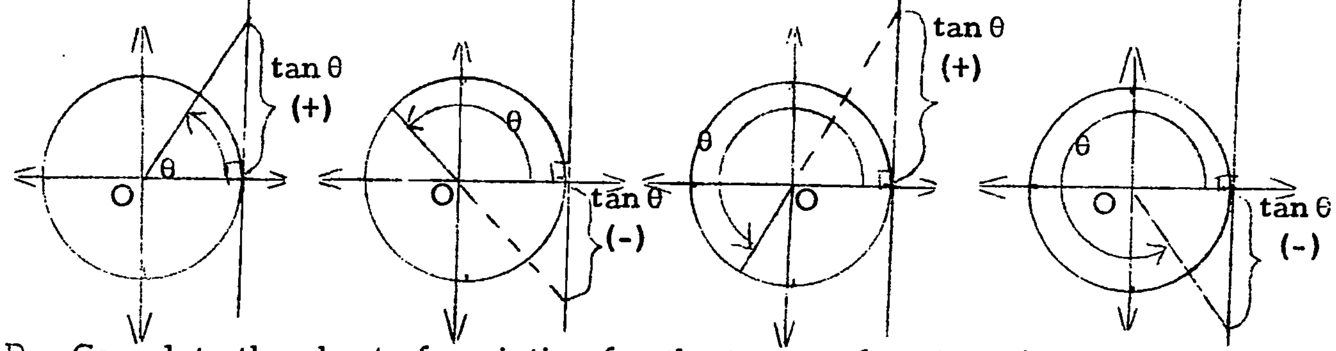


Given: Unit circle O with angle θ, point P the intersection of the unit radius vector and the unit circle.

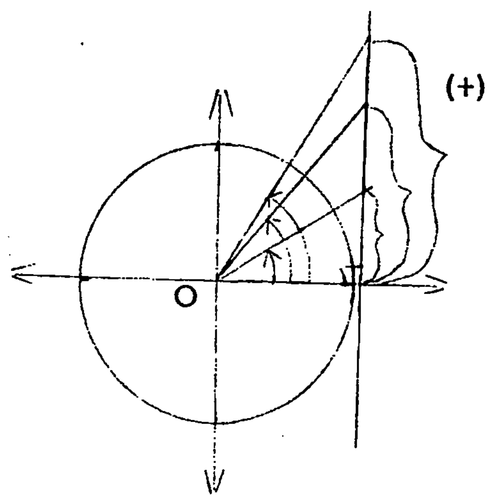
1. Draw PM $\perp$ OX
2. Draw tangent TV at point of contact T. This is the standard position of the tangent in line representation of functions
3. Extend radius vector OP intersecting TV at R
4. $\text{Tan } \theta = \dfrac{PM}{OM}$
5. $\Delta PMO \sim \Delta RTO$
6. $\dfrac{PM}{OM} = \dfrac{TR}{OT}$
7. $\text{Tan } \theta = \dfrac{TR}{OT}$
8. $\text{Tan } \theta = \dfrac{TR}{1} = TR$

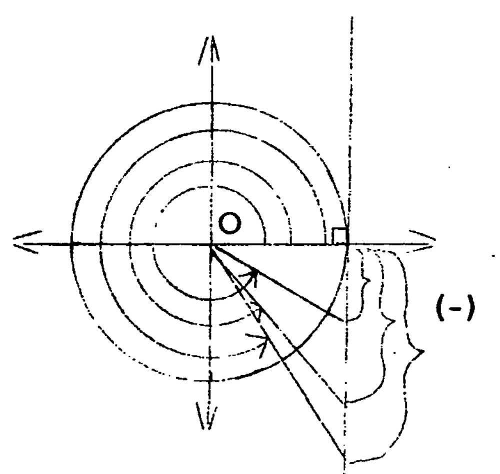
B. Definition: The line representation of the tangent value is the length of the tangent (whose point of contact is always the intersection of the circle and the positive side of the x axis) measured from the point of tangency to the point of intersection of the radius vector extended to meet the tangent. The sign of the function is positive (+) if measured above the x axis, negative (-) if measured below the x axis.

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE | PAGE
24 (cont'd)	C. Representation of the tangent function in each quadrant	

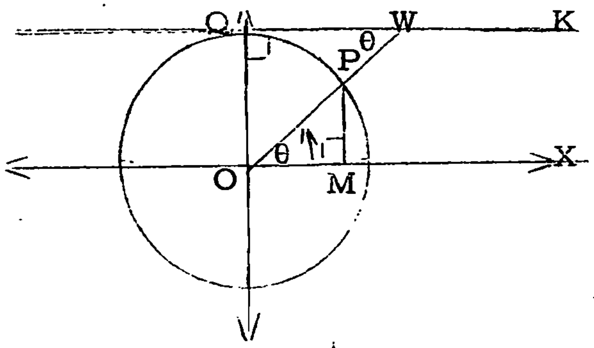


D. Complete the chart of variation for the tangent function. (Refer to chart I(D) of this unit. Note that at $\pi/2$ or $90°$ the value of tangent is undefined since it involves division by zero.) In many texts this undefined property is indicated as $\tan 90° = \pm\ \infty$. The dual positive and negative values result because $90°$ can be approached from either a positive or negative rotation, as shown below.



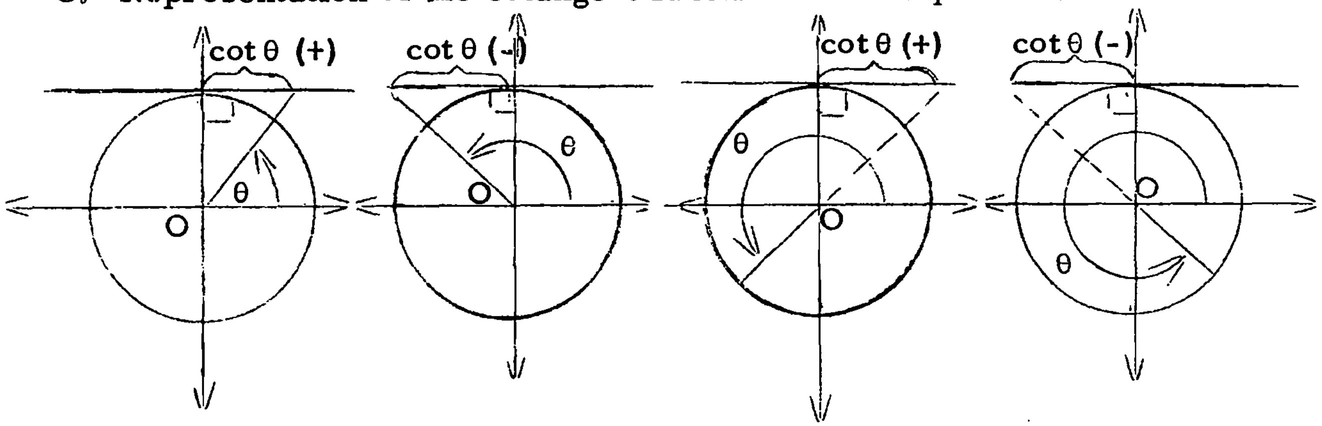


LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
24 (cont'd)	E. Craphic representation of the tangent function 1. Tangent curve: $y = \tan \theta$ 2. Period; discontinuity at $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ SUGGESTED ASSIGNMENT Repeat the classroom discussion for the line representation of cot values. Be sure to present the following ideas: A. Geometric derivation B. Definition C. Representation of the cot value in each quadrant D. Chart of variation in cotangent values E. Graphic representation of the cotangent function period, asymptotes (NOTE: The above assignment is given on pages 42 and 43 of this outline for the teacher's use.)		

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
24 (cont'd)	VI. Line representation of the cotangent function A. Geometric derivation  Given: Unit circle O with angle θ, point P the intersection of the unit radius vector and the unit circle. 1. Draw PM $\perp$ OX 2. Draw QK tangent to circle O at point of contact Q. This is the standard position of the cotangent in line representation of functions. 3. Extend radius vector OP to intersect tangent QK at W 4. $\cot\theta = \frac{OM}{MP}$ 5. $\Delta PMO \sim \Delta QWO$ 6. $\frac{OM}{MP} = \frac{QW}{OQ}$ 7. $\cot\theta = \frac{QW}{OQ}$ 8. $\cot\theta = \frac{QW}{1}$ 9. $\cot\theta = QW$	H. . . . RWM.	83 238-240 247

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
24 (cont'd)	B. Definition: The line representation of the cotangent value is the length of the tangent (whose point of contact is always the intersection of the unit circle and the positive side of the y axis) measured to the right or left from the point of contact to the intersection of the horizontal tangent and the radius vector (extended, if necessary). It is positive if measured to the right; negative, if measured to the left.		

C. Representation of the cotangent function in each quadrant



D. Complete the chart of variation for the cotangent function. (Refer to chart I (D) of this unit.)

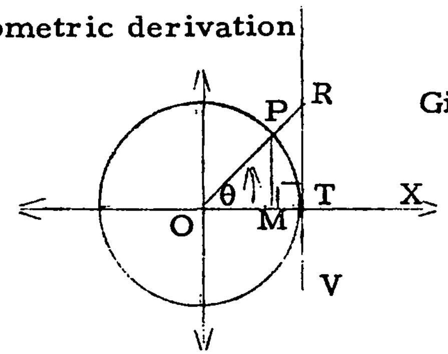
E. Graphic representation of the cotangent function

1. Cotangent curve defined by $y = \cot \theta$
2. Period; discontinuity at O and π radians

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE | PAGE
25	V. Line representation of the secant function	H. . . . 84 RWM 235-37 247

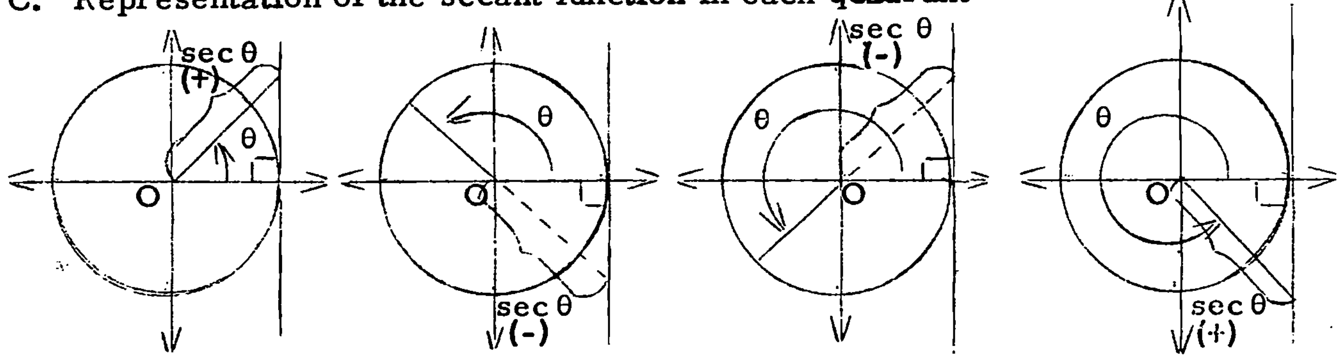
V. Line representation of the secant function

A. Geometric derivation



Given: Unit circle O with angle θ, point P the intersection of the unit radius vector and the unit circle.

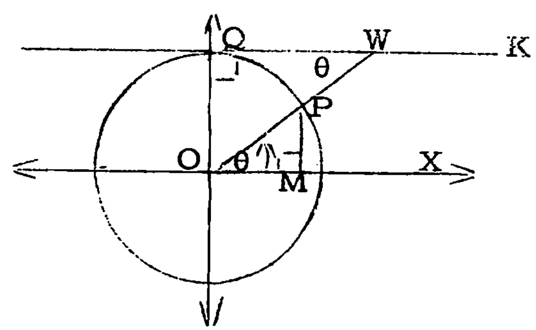
1. Draw $PM \perp OX$
2. Draw TV tangent to circle O at point of contact
3. Extend radius vector OP intersecting TV at R
4. $\sec \theta = \frac{OP}{OM}$
5. $\Delta OMP \sim \Delta RTO$
6. $\frac{OP}{OM} = \frac{OR}{OT}$
7. $\sec \theta = \frac{OR}{OT}$
8. $\sec \theta = \frac{OR}{1}$
9. $\sec \theta = OR$

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
25 (cont'd)	B. Definition: The line representation of the secant value is the length of the radius vector extended, measured from the origin to its intersection with the tangent. It is positive if measured in the same direction as the direction of the given radius vector; negative, if the radius vector must be extended through the origin in the opposite direction of the given radius vector in order to intersect the tangent in the standard position. C. Representation of the secant function in each quadrant  D. Complete the chart of variation for the tangent function. (Refer to chart I(D) of this unit.) E. Graphic representation of the secant function 1. Secant curve defined by $y = \sec \theta$ 2. Period; discontinuity at π and $\frac{3\pi}{2}$; asymptotes.		

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
25 (cont'd)	**SUGGESTED ASSIGNMENT** Repeat the classroom discussion for the line representation of the cosecant value. Be sure to include the following ideas: A. Geometric derivation B. Definition C. Representation of cosecant value in each quadrant D. Chart of variation in the cosecant values E. Graphic representation of the cosecant function; period, asymptotes; discontinuity. (NOTE: The above assignment is given on pages 46 and 47 of this outline for the teacher's use.)		

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE | PAGE
25 (cont'd)	VII. Line representation of the cosecant function	H.... 84 FSM. 61-63 RWM 238-39 247

A. Geometric derivation



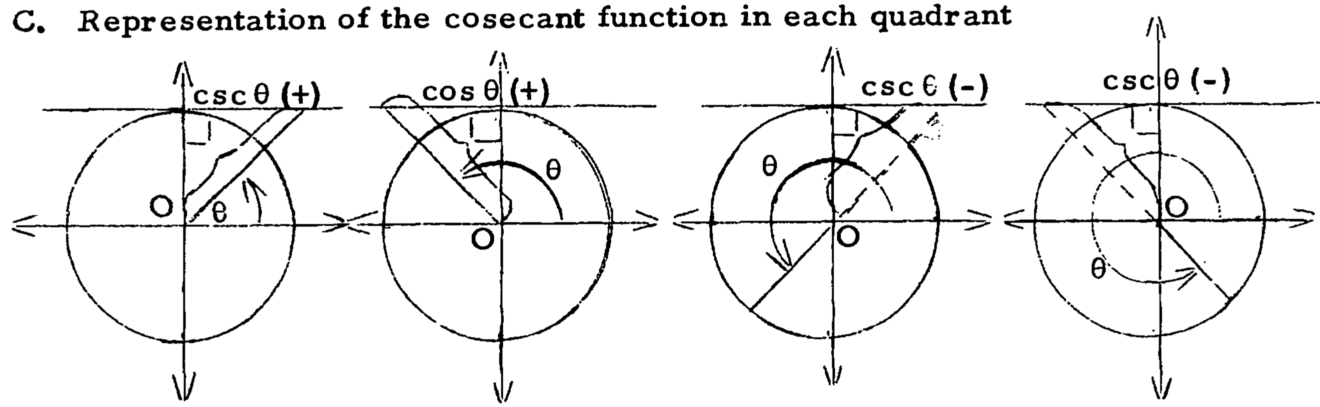
Given: Unit circle O with angle θ, point P the intersection of the unit radius vector and the unit circle.

1. Draw PM $\perp$ OX
2. QK tangent to circle O at point of contact Q
3. Extend radius vector OP to intersect tangent QK at W
4. $\csc\theta = \frac{OP}{MP}$
5. $\Delta MPO \sim \Delta QWO$
6. $\frac{OP}{PM} = \frac{OW}{OQ}$
7. $\csc\theta = \frac{OW}{OQ}$
8. $\csc\theta = \frac{OW}{1}$
9. $\csc\theta = OW$

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE | PAGE
25 (cont'd)		

B. Definition: The line representation of the cosecant value is the length of the radius vector (extended to meet the horizontal tangent) measured from the origin to the intersection of the radius vector extended and the horizontal tangent. It is positive if it is measured in the same direction as the given radius vector; negative, if the radius vector must be extended through the origin in the opposite direction of the given radius vector in order to intersect the horizontal tangent in the standard position.

C. Representation of the cosecant function in each quadrant

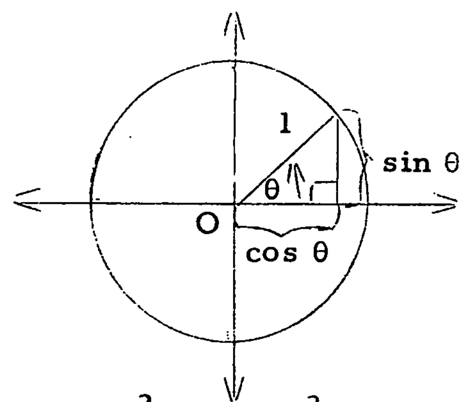


D. Complete the chart of variation for the cosecant function. (Refer to chart I(d) of this unit.)

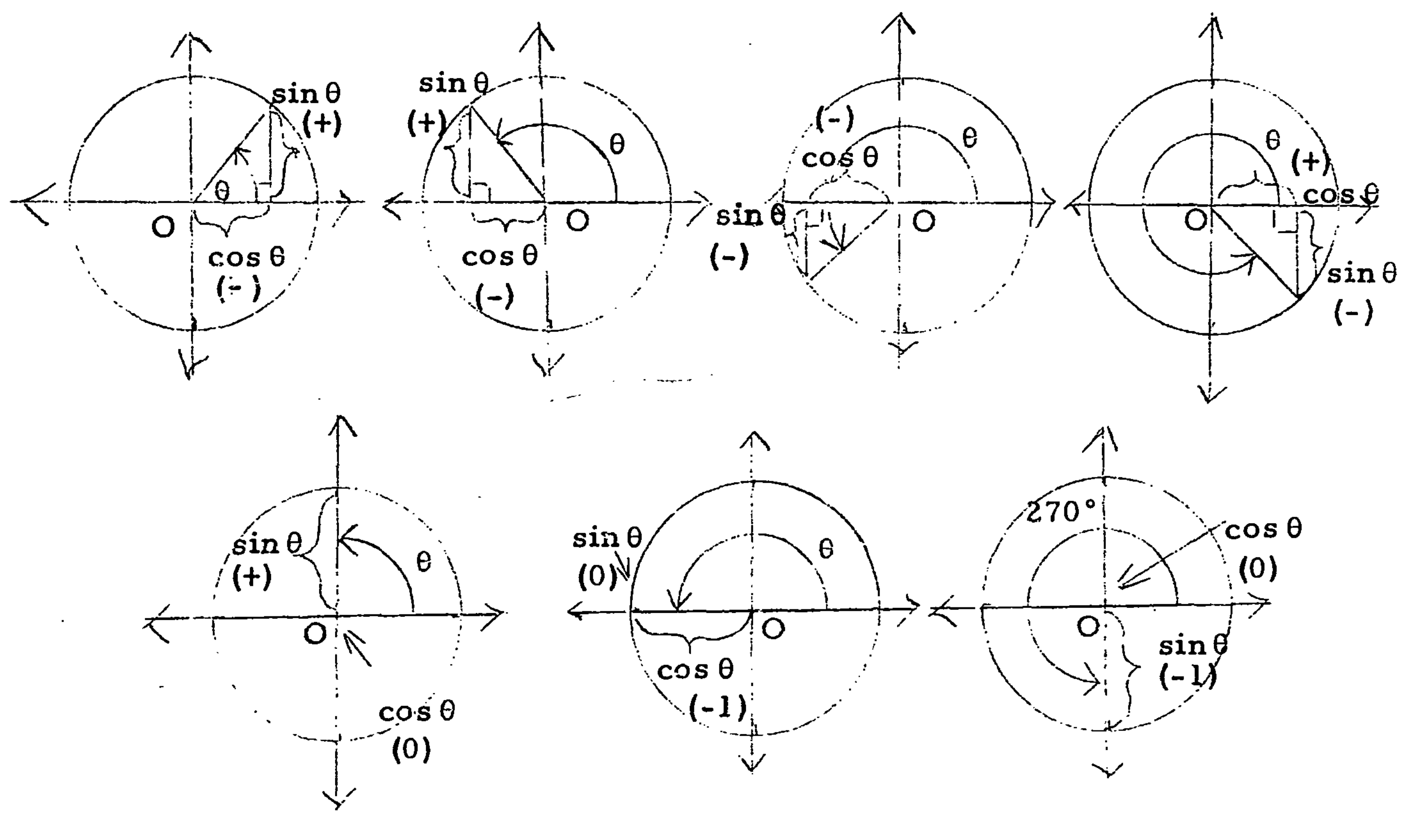
E. Graphic representation of the cosecant curve

1. Cosecant curve defined by $y = \csc \theta$
2. Period, discontinuity, asymptotes

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
26	This lesson should be devoted to further practice with line representations of functions, and sketching of graphs. SUGGESTED ASSIGNMENT H....pp. 84-86: Exercise 21, Examples 1, 2, 3, 5, 6, 7, 39, 41, 42, 43		
27	Review for test		
28	Test		

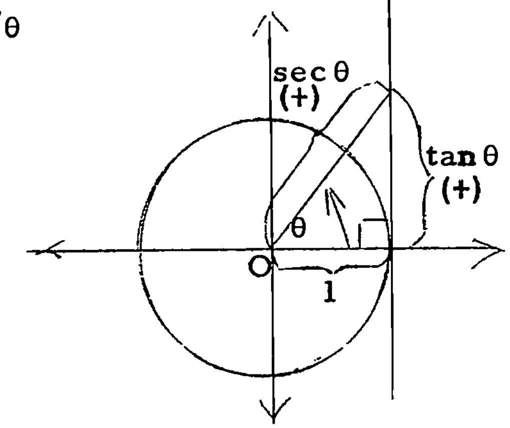
LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE | PAGE
	UNIT FOUR	
	Simple Identities and Equations	
29	I. The Fundamental Relation (3-2)	H. . . . 40-44
	The eight basic identities given on page 41 of the text may be proved by algebraic or geometric means. Since the geometric approach presents a more intuitive and graphic approach, this guide will capitalize upon the work done in the last unit on the line representations of function values. Students find these derivations simple and satisfying. The drills in this section should emphasize work with fractions and radicals.	
	II. Pythagorean Identities	
	A. $\sin^2\theta + \cos^2\theta = 1$	
	Geometric derivation	
		
	$\therefore \sin^2\theta + \cos^2\theta = 1$ by the Pythagorean Theorem	

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE | PAGE
29 (cont'd)	The line function representations are valid for the general angle of any kind of rotation, positive or negative. It can be seen that the reason for approaching the derivation of the identities geometrically first, and then showing the algebraic interpretation second is due to the fact that the geometric approach is a powerful method, capable of dealing with angles in any amount of rotation. It is wise to let the students prove these relations for themselves by considering angles in different quadrants and also for quadrantal angles. The following figures show the verity of the identity, $\sin^2\theta + \cos^2\theta = 1$, in the various quadrants and quadrantal positions.	



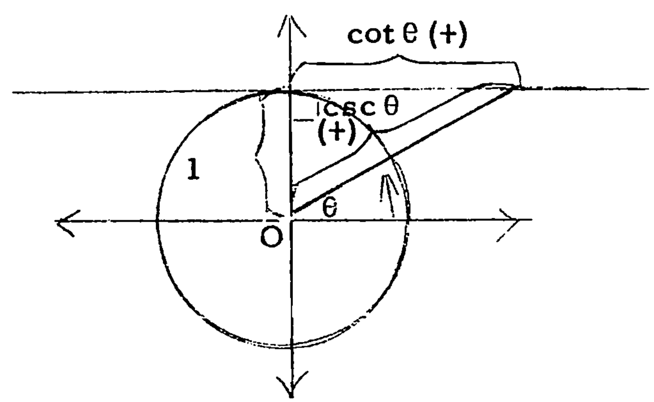
LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
29 (cont'd)			

B. $\tan^2\theta + 1 = \sec^2\theta$



The discussion of this relation should follow the pattern given in the previous section for $\sin^2\theta + \cos^2\theta = 1$

C. $\cot^2\theta + 1 = \csc^2\theta$



The discussion of this relation should follow the pattern given in the previous section for $\sin^2\theta + \cos^2\theta = 1$

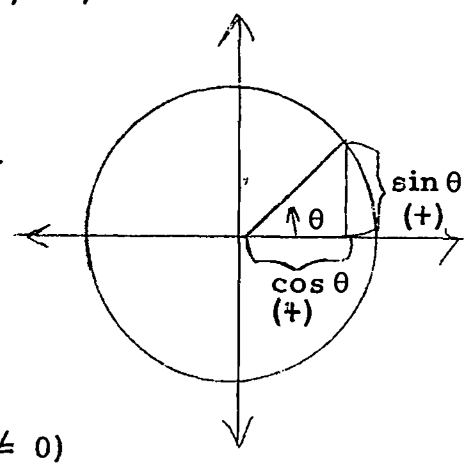
LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	PAGE
29 (cont'd)	III. Quotient identities		

A. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$; $(\cos\theta \neq 0)$

<u>Derivation</u>

1. $\tan\theta = \dfrac{\text{ordinate}}{\text{abscissa}}$

2. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$



B. $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$; $(\sin\theta \neq 0)$

<u>Derivation</u>

1. $\cot\theta = \dfrac{\text{abscissa}}{\text{ordinate}}$

2. $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

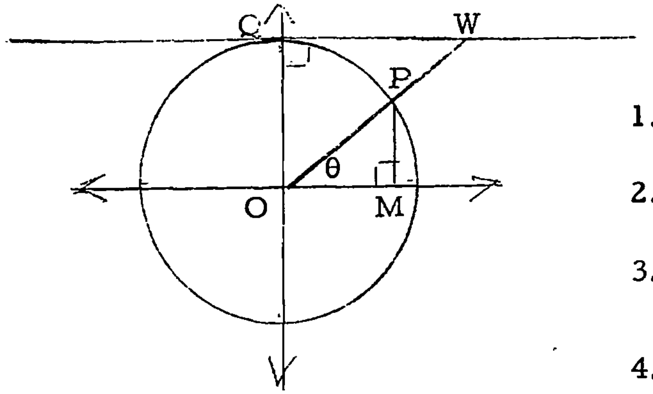
The discussion of these relations should follow the pattern given in the previous sections for $\sin^2\theta + \cos^2\theta = 1$

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE | PAGE

29 (cont'd)

IV. Reciprocal identities

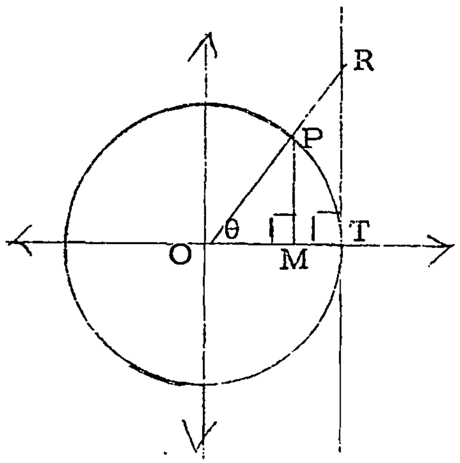
A. $\sin\theta = \dfrac{1}{\csc\theta}$ or $\sin\theta \csc\theta = 1$



<u>Derivation</u>

1. $\Delta PMO \sim \Delta QWO$ by a. a. = a. a.
2. $\dfrac{PM}{OP} = \dfrac{OQ}{OW}$
3. But $CP = OQ = 1$; $\sin\theta = PM$; $\csc\theta = OW$
4. $\therefore\ \sin\theta = \dfrac{1}{\csc\theta}$ or $\sin\theta \csc\theta = 1$

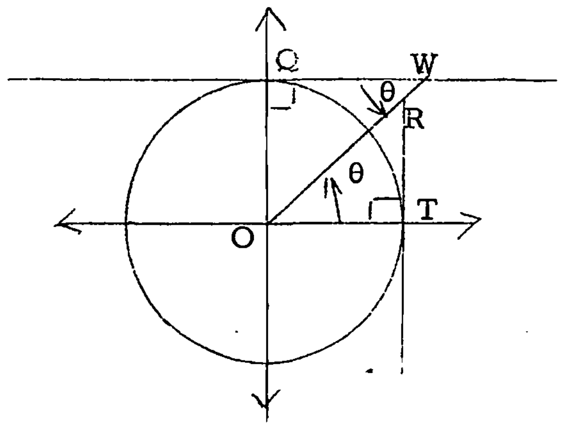
B. $\cos\theta = \dfrac{1}{\sec\theta}$ or $\cos\theta \sec\theta = 1$



<u>Derivation</u>

1. $\Delta PMO \sim \Delta RTO$ by a. a. = a. a.
2. $\dfrac{OM}{OP} = \dfrac{OT}{OR}$
3. But $OP = OT = 1$
   $\cos\theta = OM$
   $\sec\theta = OR$
4. $\therefore\ \cos\theta = \dfrac{1}{\sec\theta}$ or $\cos\theta \sec\theta = 1$

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE | PAGE
29 (cont'd)	C. $\tan\theta = \frac{1}{\cot}$ or $\tan\theta\cot\theta = 1$	



Derivation

1. $\Delta RTO \sim \Delta QWO$ by a. a. = a. a.

2. $\frac{TR}{OT} = \frac{OQ}{QW}$

3. But $OT = OQ = 1$; $\tan\theta = TR$; $\cot\theta = QW$

4. $\therefore\ \tan\theta = \frac{1}{\cot\theta}$ or $\tan\theta\cot\theta = 1$

The discussion of these relations should follow the pattern given in the previous section for $\sin^2\theta + \cos^2\theta = 1$.

V. The teacher should give an announced quiz on the statement of these eight basic identities on the following day.

SUGGESTED ASSIGNMENT

H....pp. 44, 45: Exercise 12: Examples 1, 2, 3, 5, 6, 7, 9, 10
11, 13, 14, 15, 17, 18,
19, 21, 22, 23, 29, 30

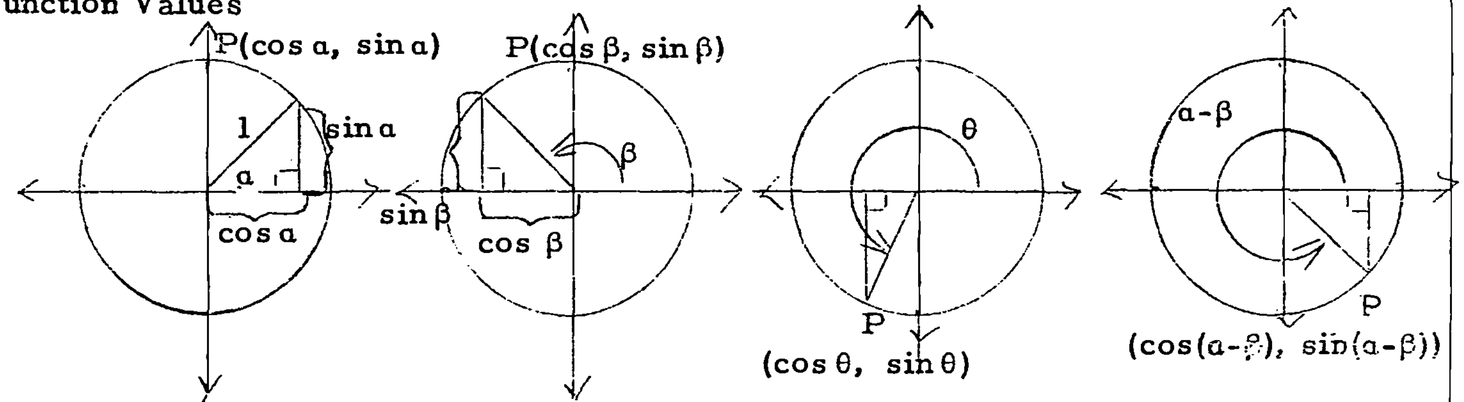
LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	PAGE
30	Algebraic Operations with the Trigonometric Functions (3-22)	H.	45-48
	SUGGESTED ASSIGNMENT		
	H....pp. 47, 48: Exercise 13: Examples 1, 2, 3, 5, 6, 7, 9, 10, 13, 14, 15		
31	Identities and Conditional Equations (3-23)	H.	48-50
	Trigonometric Identities (3-24)	H.	50-57
	SUGGESTED ASSIGNMENT		
	H....pp. 53, 54: Exercise 14: Examples 1, 2, 5, 6, 7, 9, 10, 11		
32	Trigonometric Identities (Second Day)	H.	50-57
	SUGGESTED ASSIGNMENT		
	H...p. 54:Exercise 14: Examples 13, 14, 15, 17, 18, 19, 21, 22		
33	Trigonometric Identities (Third Day)	H.	50-57
	SUGGESTED ASSIGNMENT		
	H....pp. 55-57: Exercise 14: Examples 25, 26, 33, 35, 37, 39, 45, 47		
34	Review for test		
35	Test		

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE | PAGE
	UNIT FIVE	
	Functions of Two Angles	
36	Trigonometric Function Values of Negative Angles (4-27)	H. . . . 62-64
	This text requires the derivation of trigonometric function values of negative angles so that it may be used in the derivation of sin (A + B) and cos (A + B). Teachers will observe that this sequence is reversed in the prior course of study (1960) in which the trigonometric function values of negative angles were a logical consequence of the derivations of sin (A - B) and cos (A - B). Either approach is satisfactory. However, it would be wise to use the approach in the text so that students may refer to the proofs in the text when necessary.	
	Even functions	H. . . . 63
	Odd functions	H. . . . 64
	SUGGESTED ASSIGNMENT	
	H. . . . p. 64; Exercise 17: Examples 1, 2, 3, 5, 6, 7, 8, 11, 13, 14, 18	

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE | PAGE
37	Introduction	H. . . . 87-92

The approach to the derivation of fundamental identities of composite angles using the distance formula of analytic geometry is gaining favor because of its simplicity and its application to deriving the reduction formulas and the addition and subtraction formulas for the trigonometric functions. The unit circle will still be retained. The angles used may be of any size or rotation. It will be necessary to introduce students to the labeling of coordinates by means of trigonometric functions and to review the distance formula between two points.

Labeling the Coordinates of Points on a Unit Circle by means of Trigonometric Function Values



Derivation of sin(A + B) and cos(A + B) (7-39)

SUGGESTED ASSIGNMENT

H. . . . p. 92: Exercise 22: Examples 1, 2, 3, 5, 6, 7, 9, 10, 11, 13, 14

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	PAGE
38	Reinforcement of formulas sin(A + B) and cos(A + B)	H....	87-94
	SUGGESTED ASSIGNMENT		
	H....pp. 93-94: Exercise 22: Examples 17, 18, 21, 22, 23, 25, 29, 30		
39	Derivation of Tan(A + B) (7-41)	H....	94, 95
	Derivation of Sin(A - B)		
	Derivation of Cos(A - B)		
	Derivation of Tan(A - B)		
	SUGGESTED ASSIGNMENT		
	H....pp. 96-98: Exercise 23: Examples 1, 2, 3, 5, 7, 9, 10, 11, 13		
40	Reinforcement of Lesson 39		
	SUGGESTED ASSIGNMENT		
	H....pp. 97, 98: Exercise 23: Examples 14, 17, 18, 19, 25, 27, 31, 32		
41	Double Angle Formulas (7-43)	H....	98, 99
	Half Angle Formulas (7-44)	H....	99-101
	SUGGESTED ASSIGNMENT		
	H....pp. 101-104: Exercise 24: Examples 1, 5, 6, 7, 9, 10, 11, 13, 14, 17, 21		

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
42	Reinforcement of Lesson 41 (7-43, 44) SUGGESTED ASSIGNMENT H....pp. 102-104: Exercise 24: Examples: 25, 26, 27, 29, 30, 31, 41, 42, 43, 49, 50, 55, 61	H....	90-101
43	Product to Sum Formulas; Sum to Product Formulas (7-45) SUGGESTED ASSIGNMENT H....pp. 106, 107: Exercise 25: Examples 1, 2, 3, 5, 6, 7, 9, 10, 11, 13, 15, 17	H....	104-106
44	Reinforcement of Lesson 43 (7-45) SUGGESTED ASSIGNMENT H....pp. 106, 107: Exercise 25: Examples 4, 8, 12, 14, 16, 18, 19	H....	90-101
45	Review		
46	Test		

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE | PAGE
	UNIT SIX	
	Trigonometric Equations	
47	Trigonometric Equations (8-46)	H..108-112
	Solving a Trigonometric Equation (8-46)	
	SUGGESTED ASSIGNMENT	
	H....p.112: Exercise 26: Examples 1, 2, 3, 5, 6, 7, 9, 11, 13	
48	Reinforcement of Lesson 47	
	SUGGESTED ASSIGNMENT	
	H....pp.112, 113: Exercise 26: Examples 10, 15, 17, 18, 19, 21, 23, 30	
49	Reinforcement of Lessons 47 and 48	
	SUGGESTED ASSIGNMENT	
	H....p.113: Exercise 26: Examples 31, 33, 35, 37, 39 41, 43, 45	
	Extra Credit: Study (7-42) pp. 95, 96; then do Example 47 and 49 on page 113	
50	Review	
51	Test	

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	PAGE
	UNIT SEVEN		
	LOGARITHMS		
	Introduction		
	It may well be that students have already studied some work in logarithms in Algebra II or in the Review of Academic Mathematics. In that event the teacher will wish to proceed rapidly through this unit.		
52	The Uses of Logarithms (10-53)	H. . . .	125
	Some Laws of Exponents (10-54)	H. . . .	125, 126
	Definition of Logarithm (10-55)	H. . . .	126, 127
	SUGGESTED ASSIGNMENT		
	H. . . . pp. 127, 128: Exercise 28: Odd numbered examples from 1-39		
53	Properties of Logarithms (10-56)		
	Students should be held responsible for the proofs of these theorems.		
	SUGGESTED ASSIGNMENT		
	H. . . . pp. 129, 130: Exercise 29: Odd numbered examples from 1-39		

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
54	~~Systems of Logarithms (10-57)~~	H....	130-134
	Characteristic and Mantissa (10-58)		
	Method of Determining Characteristics (10-59)		
	SUGGESTED ASSIGNMENT		
	H....p.134: Exercise 30: Odd numbered examples 1-27		
55	A Five-Place Table of Mantissas (10-60)	H....	135
	Given N, To Find Log N (10-61)	H....	135
	Given Log N, To Find N (10-62)	H....	135, 136
	SUGGESTED ASSIGNMENT		
	H....p.136: Exercise 31: Odd numbered examples 1-23		
56	Interpolation (10-63)	H....	136-139
	SUGGESTED ASSIGNMENT		
	H....pp.138,139: Exercise 32: Examples 1, 5, 9, 13, 15, 17, 21, 25, 29, 31		
57	Logarithmic Computation (10-64)	H....	139-141
	SUGGESTED ASSIGNMENT		
	H....pp.141-143: Exercise 33: Examples 3, 5, 9, 13, 14, 17, 19		

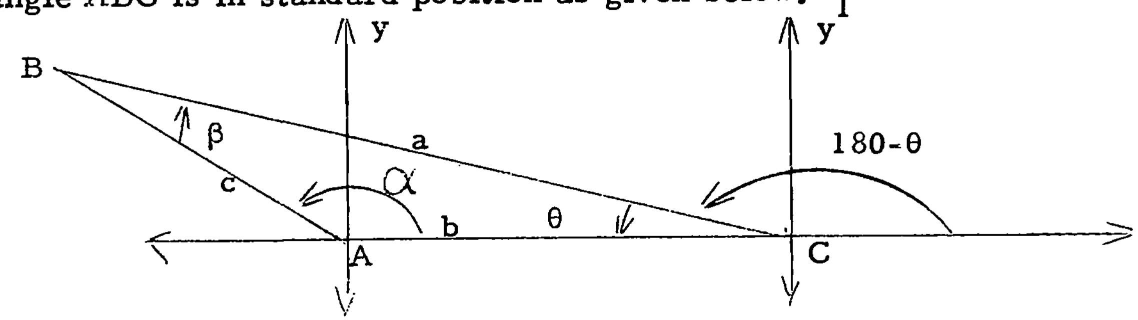
LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
58	Reinforcement of Lesson 57		
	SUGGESTED ASSIGNMENT		
	H.... p. 142: Exercise 33: Examples 33, 37, 40, 42, 43		
59	Logarithmic Equations (10-66)	H....	144-146
	Exponential Equations (10-67)	H....	146
	Change of Base of Logarithms (10-68)	H....	146, 147
	SUGGESTED ASSIGNMENT		
	H.... pp. 147, 148: Exercise 34: Examples 1, 5, 9, 11, 13, 17, 19, 23		
60	Review		
61	Test		

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	PAGE
	UNIT EIGHT		
	Logarithmic Solution of Right Triangles		
62	Logarithms of Trigonometric Functions (11-69)	H....	149-150
	SUGGESTED ASSIGNMENT		
	H....pp. 150, 151: Exercise 35: Examples 1, 5, 9, 13, 17, 21, 25, 29, 33, 37		
63	Logarithmic Solution of Right Triangles (11-70)	H....	151-154
	SUGGESTED ASSIGNMENT		
	H....p. 154: Exercise 36: Examples 1, 2, 3, 5, 9, 13, 17		
64	Vectors (11-71)	H....	154-159
	SUGGESTED ASSIGNMENT		
	H....1, 3, 9, 10, 11		
65	Review		
66	Test		

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE | PAGE
	UNIT NINE	
	Oblique Triangles	H....160
67	Introduction (12-72)	
	The Law of Sines (12-73)	H....160-165

The derivation of the Law of Sines as presented in the text is traditional, and entirely satisfactory. However, since the course is to emphasize the analytic approach, the following derivation is presented for the teacher.

Choose a rectangular coordinate system so that the angle α (or $\angle CAB$) of triangle ABC is in standard position as given below.



1. The coordinates of B are $(c \cos\alpha, c \sin\alpha)$

2. If, however, the origin of the coordinate system is at C with angle $[180-\theta]$ in standard position then the coordinates of B are

$$(a \cos[180-\theta], a \sin[180-\theta])$$

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
67 (cont'd)	3. Since B is the same distance above the x-axis in either case, then the second coordinates of B are equal. Therefore, $c \sin \alpha = a \sin [180-\theta]$ 4. But $\sin [180-\theta] = \sin \theta$ 5. $\therefore\ c \sin \alpha = a \sin \theta$ 6. Dividing each member of (5) by $\sin \alpha \sin \theta$, we obtain $\frac{c}{\sin \theta} = \frac{a}{\sin \alpha}$ 7. If the coordinate axes are chosen so that the origin is considered first at A with α in the standard position and then the origin is considered next at B with α in the standard position, the same argument yields. $\frac{b}{\sin \beta} = \frac{a}{\sin \alpha}$ 8. Hence $\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \theta}$ SUGGESTED ASSIGNMENT H....pp. 164, 165∠ Exercise 38∠ Examples 3, 6, 9, 12, 18		

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
68	The Ambiguous Case: SSA (12-75) SUGGESTED ASSIGNMENT H.... pp. 169, 170.: Exercise 39: Examples 1, 3, 5, 7, 9	H....	165-170
69	The Law of Cosines (12-76) The proof of the Law of Cosines as developed in the text applies only to triangles. It is a mistake to think that the Law of Cosines as being restricted only to triangles. Indeed, it is one of the most important identities of trigonometry. It is also useful in further investigations and development of trigonometry. For example, the formula of sin (A - B) can be derived by using the Law of Cosines. The teacher should present the derivation of this law as given on the next page. The proof is analytic in form and holds for angles of <u>ANY</u> measure, including, of course, the case when the angle is less than 180°, and thus may be an angle of a triangle.	H....	165-170

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE | PAGE
69 (cont'd)	To emphasize the dual role of the Law of Cosines, two figures with identical lettering are used. Each step of the derivation applies to either figure. In Figure I, θ is an obtuse angle in standard position. In Figure II, θ is an angle in Quadrant III in standard position.	

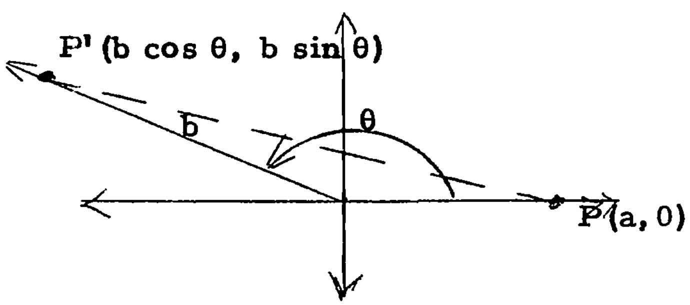


FIGURE I

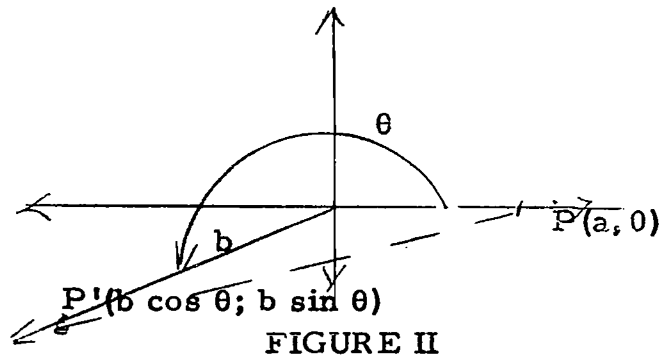


FIGURE II

1. Let P be a point on the initial side of θ at a distance of $\underline{a}$ from the origin. Its coordinates are, therefore, (a, 0)

2. Let P' be a point on the terminal side of θ at a distance of $\underline{b}$ from the origin. Its coordinates are ($b \cos \theta$, $b \sin \theta$).

3. Now if PP' were drawn in Figure I, there would be formed a $\Delta PP'0$ in which sides a and b include the angle θ. However, this would not be true for Figure II since no angle of a triangle can exceed 180°.

4. The distance formula is usually stated as

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

or as

$$d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$$

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	PAGE
69 (cont'd)	5. Substituting the coordinates of P and P' in this latter formula, $PP'^2 = (b \cos \theta - a)^2 + (b \sin \theta - 0)^2$ $= b^2 \cos^2 \theta - 2ab \cos \theta + a^2 + b^2 \sin^2 \theta$ $= b^2 (\sin^2 \theta + \cos^2 \theta) + a^2 - 2ab \cos \theta$ $PP'^2 = b^2 + a^2 - 2ab \cos \theta$ LAW OF COSINES If θ is an angle in standard position and if a and b represent the respective distances from the origin of two points on the sides of the angle θ, then the square of the distance d between these two points is given by the formula: $d^2 = a^2 + b^2 - 2ab \cos \theta.$ If the Law of Cosines is applied to any triangle ABC with sides a, b, c, then the cosine law may be written as $a^2 = b^2 + c^2 - 2bc \cos A$ $b^2 = a^2 + c^2 - 2ac \cos B$ $c^2 = a^2 + b^2 - 2ab \cos C$		

<table>
<tr><th>LESSON</th><th>SCOPE AND TEACHING SUGGESTIONS</th><th>REFERENCE
CODE | PAGE</th></tr>
<tr><td>69
(cont'd)</td><td>Thus, the Law of Cosines for triangles may be stated as:

The square of any side of a triangle equals the sum of the squares of the other two sides minus twice the product of those two sides and the cosine of the included angle.

The formulas as stated above are useful in finding the length of a side when the lengths of two sides and the measure of the included angle are given. If each of the formulas are transformed as

$$\text{Cos } C = \frac{a^2 + b^2 - c^2}{2ab}$$
$$\text{Cos } B = \frac{a^2 + c^2 - b^2}{2ac}$$
$$\text{Cos } A = \frac{b^2 + c^2 - a^2}{2bc}$$

then the measure of the included angle may be found when the lengths of the three sides of a triangle are known.

Applications of the Law of Cosines: SAS and SSS (12-77)

SUGGESTED ASSIGNMENT

H....pp. 173, 174: Exercise 40: Examples 1, 5, 9, 13, 15</td><td>

H....171-173</td></tr>
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LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
70	Summary (12-78)	H....	174, 175
	The Area of a Triangle (12-79)	H....	175, 176
	SUGGESTED ASSIGNMENT		
	H....p.177: Exercise 41: Examples 1, 3, 5, 7, 9, 11		
71	Review		
72	Test		

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE | PAGE
	UNIT TEN	
	Inverse Trigonometric Functions	
73	Inverse Trigonometric Functions (13-80)	H.... 182-183
	Principal Values of the Inverse Trigonometric Functions (13-81)	RWM. 253-270
		M.... 284-294

Sin, cos, tan, cot, sec, and csc are all functions as defined in Lesson 6 since the ordered pairs in each function have different first coordinates. If the ordered pairs in any function, say the sin, are interchanged, some of the ordered pairs would be

$$(\frac{1}{2}, 30°), (\frac{1}{2}, 150°), (\frac{1}{2}, 390°), (\frac{1}{2}, 510°), \text{ etc.}$$

The relation consisting of these interchanged ordered pairs is <u>not</u> a function. Difficulty arises when this relation of these interchanged ordered pairs is called an inverse, because we would like to reserve the concept of "inverse" for functions. It is much clearer to students if the following ideas are used:

(1) The <u>converse</u> of a function R is the relation obtained by interchanging the first and second coordinates in each ordered pair of R.

(2) The converse of a function R may or may not be a function.

(3) The converse of a function is the <u>inverse</u> of R if and only if the converse is also a function.

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
73 (cont'd)	(4) To test whether the converse of a function is also the inverse, we may use either of (1) the ordered pair test (2) the vertical line test The graphs of the inverse functions in the text are not typical of most texts. Teachers should refer to the RWM reference for the graphs. The vertical line test applied to these graphs will show that the converse of every trigonometric function is <u>not</u> a function. However, when the domain of the trigonometric function is restricted, then the converse of the restricted function becomes a function, and hence an inverse trigonometric function. The principal values mentioned in the text are chosen in such a way that the converse becomes an inverse. This explanation is seldom seen in most texts. Thus the idea of "**Arcsin**" is involved with the converse of sin, while the "Arcsin" is involved with the inverse of the restricted sin function. Stress the idea that in finding the value of arcsin $(\frac{1}{2})$, we are simply trying to find the second coordinate of the ordered pair $(\frac{1}{2})$, which belongs to the <u>converse</u> of the sin function. However, in finding the value of Arcsin $(\frac{1}{2})$, we are trying to find the second coordinate of the ordered pair $(\frac{1}{2})$ of the <u>inverse</u> of the restricted sin function. SUGGESTED ASSIGNMENT H....p.185: Exercise 43: Examples 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21		

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
74	Operations Involving Inverse Trigonometric Functions (13-82) SUGGESTED ASSIGNMENT H....pp. 187, 188: Exercise 44: Examples 1, 3, 5, 7, 9, 11, 13, 15, 17, 19	H....	186-189
75	Inverse Functions (13-83) (Optional) or Review The teacher should omit this section if the students are not familiar with composite functions or the operation of composition. If the teacher does not present this, then the lesson should be used for review. SUGGESTED ASSIGNMENT H....pp. 190, 191: Exercise 45: Examples 1, 2, 3,	H.... M....	190 280, 281, 298
76	Test		

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
	UNIT ELEVEN		
	Complex Numbers		
77	Complex Numbers (14-84)	H. . . .	192-195
	This lesson should be a rapid review since students have studied this topic in earlier courses.		
	SUGGESTED ASSIGNMENT		
	H. . . . p. 195: Exercise 46: Examples Odd numbers problems from 1-25		
78	Graphical Representation of Complex Numbers (14-85)	H. . . .	196
	Graphical Addition of Complex Numbers (14-85)	H. . . .	197
	Trigonometric Form of a Complex Number (14-87)	H. . . .	197-200
	SUGGESTED ASSIGNMENT		
	H. . . . pp. 199, 200: Exercise 47: Odd numbers problems from 1-27		
79	Multiplication of Complex Numbers in Trigonometric Form (14-88)	H. . . .	200, 201
	DeMoivre's Theorem (14-89)	H. . . . RWM.	201 277-288
	Roots of Complex Numbers (14-90)	H. . . .	201, 202

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
79 (cont'd)	SUGGESTED ASSIGNMENT H....pp. 203, 204: Exercise 48: Examples, 1, 3, 5, 7, 9, 13, 17, 21, 23, 25, 34		
80	Review		
81	Test For the final examination, it is recommended that you use the Cooperative Standardized Test in Trigonometry. You should ask your department chairman to procure these from the Testing Department at Alleghany Annex at least a month before finals. Scoring keys and norms are available also. This test requires about 45 minutes. Many teachers like to give this standardized test as one of a two part examination. The second part might include problems which require more applications and computations.		

APPENDIX I

SPECIAL RIGHT TRIANGLES

Certain right triangles appear frequently in problems of the physical world such as engineering and in problems of related mathematics courses such as trigonometry. These special right triangles may be classified into sets of triangles, each set containing only triangles that are similar to one another. You should be able to recognize the special right triangles discussed in this unit, to understand the relationships that exist for each set, and to apply these relationships in the solution of problems.

I. The 3, 4, 5 right triangles.

Since $3^2 + 4^2 = 5^2$, the converse of the Pythagorean Theorem tells us that a triangle whose sides have measures 3, 4, 5 is a right triangle. In a similar manner, we can conclude that if any positive number k is used as a constant of proportionality, then a triangle whose sides measure 3k, 4k, 5k is a right triangle. For example, any triangle whose sides measure 6, 8, 10 or 150, 200, 250 is a member of this set of right triangles.

Example 1. Find the length of the hypotenuse of a right triangle if the length of the two legs of the triangle are 18 and 24.

You should note that the triangle is a 3, 4, 5 right triangle with 6 as the constant of proportionality since $18 = 6 \cdot 4$. Therefore, the length of the hypotenuse of the triangle is $6 \cdot 5$ or 30.

Example 2. Triangle ABC is a right triangle with $\angle$ C as the right angle. AC = 75 and BC = 45. Find the length of AB.

Since the length of AC is $15 \cdot 5$ and the length of BC is $15 \cdot 3$, the triangle is a 3, 4, 5 triangle with 15 as the constant of proportionality. Therefore, AB must equal $15 \cdot 4$ or 60.

II. The 5, 12, 13 right triangles.

Explain why a triangle whose sides have measures of 5, 12, 13 is a right angle. Explain why a triangle whose sides have measures of 5k, 12k, and 13k is a right triangle if k is some positive number. Explain why all these triangles are similar. Give other examples of three numbers which are the respective measures of the sides of triangles in this set of similar right triangles.

Example 1. Find the length of the hypotenuse of a right triangle if the lengths of the two legs of the triangle are 15 and 36.

You should note that the triangle is a 5, 12, 13 right triangle with 3 as the constant of proportionality since $15 = 3 \cdot 5$ and $36 = 3 \cdot 12$. Therefore, the length of the hypotenuse of the right triangle is $3 \cdot 13$ or 39.

Example 2. Triangle ABC is a right triangle with $\angle$ C as the right angle. If the hypotenuse of the triangle has a measure of 78 and one of the legs of the triangle has a measure of 72, find the measure of the other leg of the triangle.

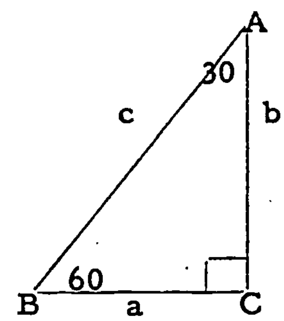
Since $78 = 6 \cdot 13$ and $72 = 6 \cdot 12$, the triangle is a 5, 12, 13 triangle with 6 as the constant of proportionality. Therefore, the measure of the other leg is $6 \cdot 5$ or 30.

III. The 30, 60, 90 triangles.

Unlike the sets of triangles in (I) and (II), the triangles in this set are usually designated by measures of angles rather than lengths of sides. Suppose that in $\triangle ABC$, $m\angle C = 90$, $m\angle B = 60$, $m\angle A = 30$. We know by Theorem 5 · 7 on page 202 of your text, that the length of BC is one-half the length of AB.

If we let $m(BC) = a$, then $m(AB) = c = 2a$, and by the Pythagorean Theorem

$$a^2 + b^2 = (2a)^2$$
$$a^2 + b^2 = 4a^2$$
$$b^2 = 3a^2$$
$$b = a\sqrt{3}$$



Thus we see that if $\triangle ABC$ is a 30, 60, 90 triangle $(a, b, c) \overset{=}{p} (1, \sqrt{3}, 2)$.

We can also prove the converse of this statement. If $\triangle A'B'C'$ is a triangle with $(B'C', C'A', A'B') \overset{=}{p} (1, \sqrt{3}, 2)$, then $\triangle A'B'C'$ is a 30, 60, 90 triangle.

We know that $\triangle A'B'C'$ is similar to $\triangle ABC$ by the SSS Similarity Theorem. Therefore, $m\angle A' = 30$, $m\angle B' = 60$, and $m\angle C' = 90$ because corresponding angles of similar triangles are equal in measure.

The above discussion can be summarized in the following theorem and corollaries:

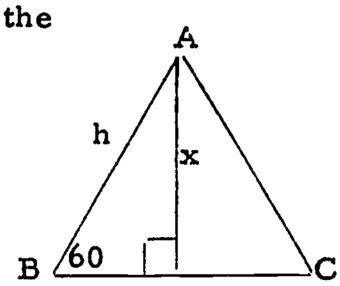
Theorem A. Triangle ABC is a right triangle with $m\angle A = 30$, $m\angle B = 60$, $m\angle C = 90$ if and only if $(a, b, c) \overset{=}{p} (1, \sqrt{3}, 2)$.

Corollary 1. In a right triangle whose acute angles have measures of 30 and 60, the shorter leg is one-half the hypotenuse. $(s_{30} = \frac{h}{2})$

Corollary 2. In a right triangle whose acute angles have measures of 30 and 60, the longer leg is equal to one-half the hypotenuse multiplied by $\sqrt{3}$. $(s_{60} = \frac{h\sqrt{3}}{2})$

Example 1. Find the length of the altitude of an equilateral triangle if the length of one of the sides is 8.

The altitude of an equilateral triangle bisects the vertex angle making a 30, 60, 90 triangle.



$(4, x, 8) \overset{=}{p} (1, \sqrt{3}, 2)$.

The constant of proportionality is 4.

Therefore, $x = 4\sqrt{3}$.

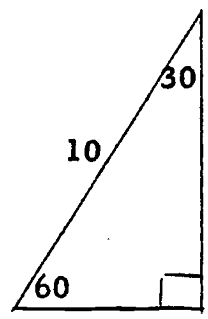
The problem can also be solved by using Corollary 2.

$$s_{60} = \frac{h\sqrt{3}}{2}$$

$$s_{60} = \frac{8 \cdot \sqrt{3}}{2}$$

$$s_{60} = 4\sqrt{3}.$$

Example 2. Find the measures of the sides of a 30, 60, 90 triangle if the length of the longer leg of the triangle is 10.



$(a, 10, c) \overset{=}{p} (1, \sqrt{3}, 2)$.

The constant of proportionality is $\frac{10}{\sqrt{3}}$.

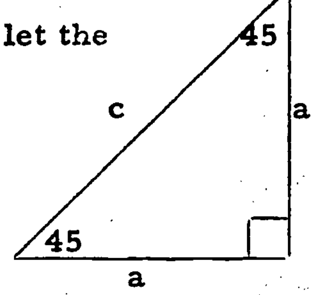
Therefore, $a = \frac{10}{\sqrt{3}}$ and $c = \frac{2 \cdot 10}{3}$, or

$$a = \frac{10\sqrt{3}}{3} \text{ and } c = \frac{20\sqrt{3}}{3}.$$

IV. The 45, 45, 90 triangles.

Since the triangles in this set have two angles that are equal in measure, they are sometimes called isosceles right triangles. We can prove that the lengths of the sides of any isosceles right triangle are proportional to $(1, 1, \sqrt{2})$.

In $\triangle ABC$. $m\angle C = 90$, $m\angle A = m\angle B = 45$. If we let the length of AC = a, then the length of BC = a. Why? By the Pythagorean Theorem



$$c^2 = a^2 + a^2$$

$$c^2 = 2a^2$$

$$c = a\sqrt{2}.$$

Therefore, we see that if $\triangle ABC$ is a 45, 45, 90 triangle, $(a, a, c) \overset{=}{p} (1, 1, \sqrt{2})$.

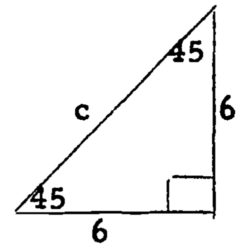
We shall now prove the converse of this statement. If $\triangle A'B'C'$ is a triangle with $(a', b', c') \overset{=}{p} (1, 1, \sqrt{2})$, then $\triangle A'B'C'$ is a 45, 45, 90 triangle.

We know that $\triangle A'B'C'$ is similar to $\triangle ABC$ by SSS Similarity Theorem. Therefore, $m \angle C = 90$, $m \angle A = 45$, $m \angle B = 45$ because corresponding angles of similar triangles have equal measure.

The above discussion can be summarized in the following theorem and corollary.

> <u>Theorem B</u>. Triangle ABC is a right triangle with $m \angle A = m \angle B = 45$ and $m \angle C = 90$ if and only if $(a, b, c) \overset{=}{p} (1, 1, \sqrt{2})$.
>
> <u>Corollary.</u> In an isosceles right triangle, either leg of the triangle is equal to one-half the hypotenuse multiplied by $\sqrt{2}$. $\left(s_{45} = \frac{h\sqrt{2}}{2}\right)$

<u>Example 1.</u> Find the length of the hypotenuse of a right isosceles triangle if each leg of the triangle has a measure of 6.

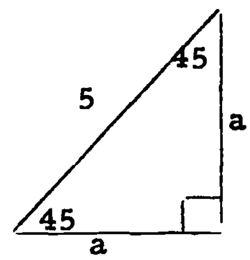


$(6, 6, c) \overset{=}{p} (1, 1, \sqrt{2})$

The constant of proportionality is 6.

Therefore, $c = 6\sqrt{2}$.

Example 2. Find the length of each leg of an isosceles right triangle if the length of the hypotenuse is 5.



$(a, a, 5) \overset{=}{p} (1, 1, \sqrt{2})$

The constant of proportionality is $\frac{5}{\sqrt{2}}$.

Therefore, $a = \frac{5}{\sqrt{2}}$, or $\frac{5\sqrt{2}}{2}$.

The problem can also be solved by using the corollary.

$$s_{45} = \frac{h\sqrt{2}}{2}$$

$$s_{45} = \frac{5\sqrt{2}}{2}.$$

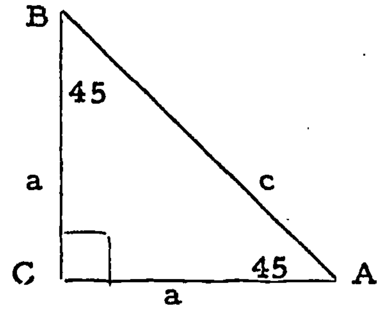
Problem Set

1. In each of the following exercises, the lengths of a leg and the hypotenuse of a right triangle are given. Which of the measures belong to a triangle similar to the 3, 4, 5 triangle? to the 5, 12, 13 triangle? to the 1, $\sqrt{3}$, 2 triangle? to the 1, 1, $\sqrt{2}$ triangle?

(a) 6, 10	(e) 3, 6	(i) 6, $6 \cdot 5$
(b) 12, 15	(f) 3, $2\sqrt{3}$	(j) 24, 26
(c) 24, 25	(g) 8, 10	(k) $3\sqrt{2}$, $5\sqrt{2}$
(d) 15, 39	(h) 1.5, 2.5	(l) $\sqrt{2}$, 2

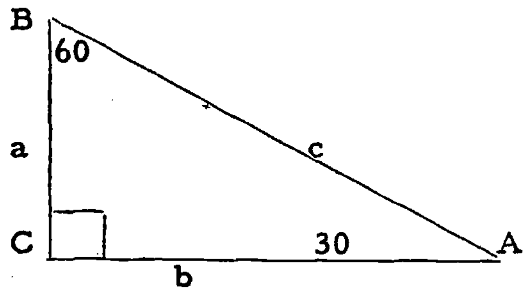
2. In each part of Problem 1, find the length of the side which is not given.

3. Complete the table for the right isosceles triangle in the diagram.



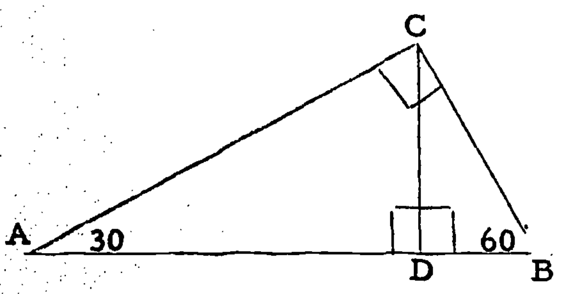
	a	c
(a)	10	
(b)	5	
(c)		9
(d)		6
(e)		$3\sqrt{2}$
(f)	$5\sqrt{2}$	

4. Complete the table for the 30-60-90 triangle in the diagram.



	a	b	c
(a)	10		
(b)	5		
(c)		18	
(d)		$9\sqrt{3}$	
(e)			20
(f)			$12\sqrt{3}$

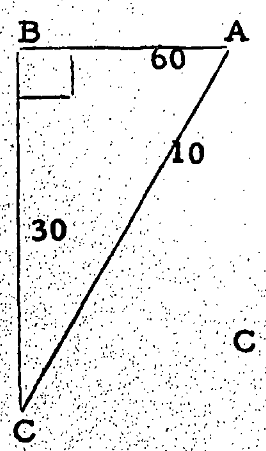
5. In each of the following exercises the length of one segment in the adjacent plane figure is given. Find the lengths of the remaining segments.



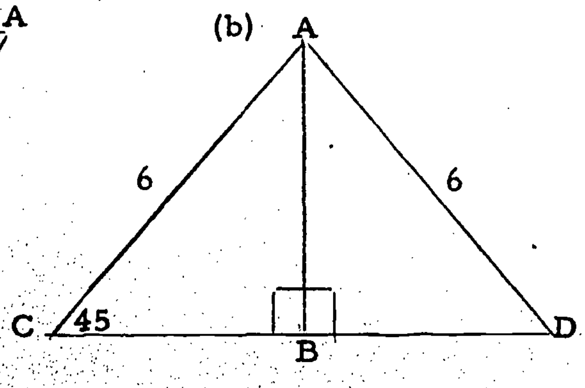
	m(AB)	m(BC)	m(CD)	m(AD)	m(DB)	m(AC)
(a)	8	____	____	____	____	____
(b)	____	2	____	____	____	____
(c)	____	____	$4\sqrt{3}$	____	____	____
(d)	____	____	____	9	____	____
(e)	____	____	____	____	$10\sqrt{3}$	____
(f)	____	____	____	____	____	$8\sqrt{3}$

6. Use the three corollaries in this unit to find the length of AB and BC in each of the following plane figures. You should do all work mentally and write only the answer.

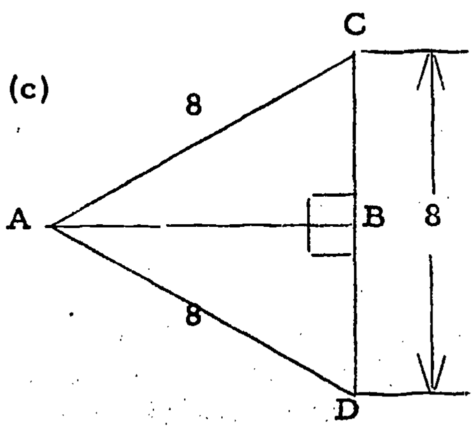
(a)



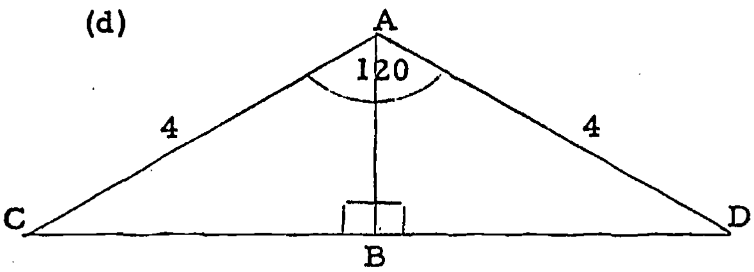
(b)



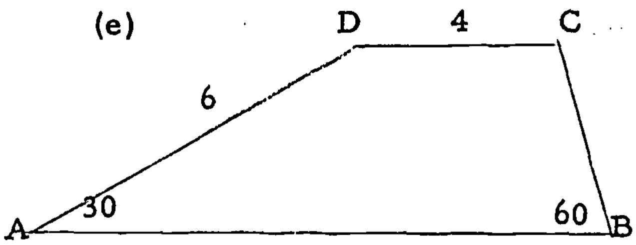
(c)

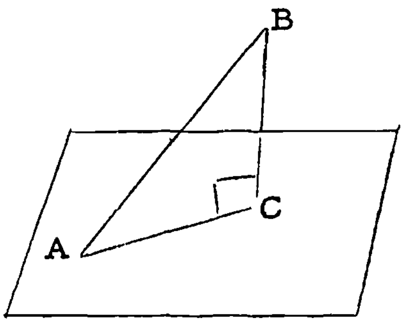


(d)



(e)





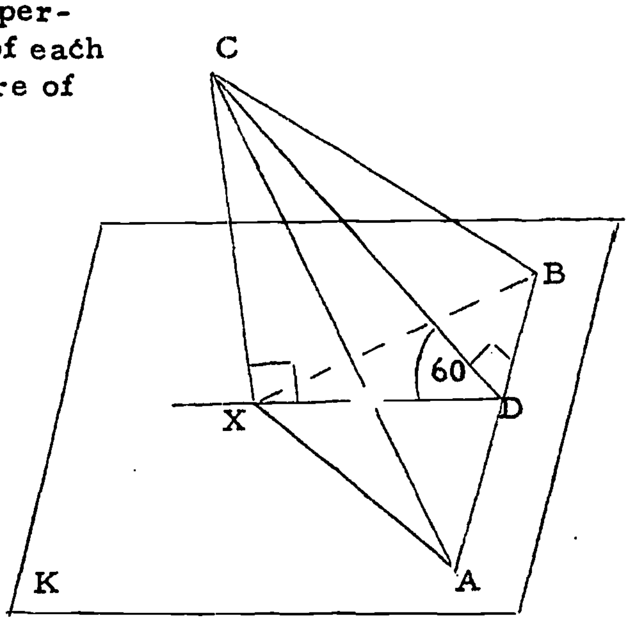
7. In the diagram, BC $\perp$ AC and m$\angle$A = 30.

   (a) If m(AB) = 6, find m(AC).
   (b) If m(AB) = 4 $\sqrt{3}$, find m(AC).
   (c) If m(AC) = 9, find m(AB).
   (d) If m(AC) = 6 $\sqrt{3}$, find m(AB).

8. Repeat problem 7 if m$\angle$A = 60.

9. In the diagram, $\triangle$ABC is an equilateral triangle which is inclined at an angle to plane K. The measure of the dihedral angle C-AB-X is 60; CX is perpendicular to plane K; CD is perpendicular to AB. Find the measure of each of the following segments if the measure of AB = 6.

(a)	AC	(d)	CX
(b)	BC	(e)	AX
(c)	CD	(f)	BX



10. Repeat problem 9 if the measure of the dihedral angle is 30; (b) 45.

BALTIMORE COUNTY PUBLIC SCHOOLS

APPENDIX II

REVIEW OF ELEMENTARY SET CONCEPTS

by

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Supervisor of Senior High School Mathematics

July 1965

REVIEW OF ELEMENTARY SET CONCEPTS

1. Member — Each object in a set is called a <u>member</u> (or <u>element</u>) of the set.

2. The symbol " $\in$ " means "belongs to" or "is a member of" or "is an element of."

3. Set Notation

   (1) The <u>phrase</u> method

   Example: the set of the (names of the) vertices of $\triangle ABC$

   (2) The <u>roster</u> (listing) method

   Example: Using the same set as in (1) above: $\{ A, B, C \}$

   (3) The <u>rule</u> method

   Example: Using the same set as in (1) and (2) above:

   $$\{ x \mid x \text{ is a vertex of } \triangle ABC \}$$

   This is read as "the set of <u>all</u> x such that x is a vertex of triangle ABC."

4. Infinite set — A set which is unending is called an <u>infinite set.</u>

5. Finite set — A set in which the members can be listed and the listing terminates is called a <u>finite set.</u>

6. The empty set — The set which contains no members is called the <u>empty</u> set or the <u>null</u> set.

   The symbol for the empty set is " ϕ "

7. Equal sets — Two sets are <u>equal</u> if and only if they have the same members.

8. Subset — Set A is a subset of set B (denoted "$A \subset B$") if and only if each member of A is also a member of B.

   Every set is a subset of itself; that is, if A is a set, then $A \subset A$.

   The empty set is a subset of <u>every</u> set; that is, if A is a set, then $\phi \subset A$.

   Set A = set B if and only if $A \subset B$ and $B \subset A$.

Set A is a proper subset of set B if and only if there is at least one member of B which is not a member of A.

Examples: Let $A = \{1, 3, 4\}$
$B = \{1, 2, 3, 4, 5\}$
$C = \{4\}$
$D = \{2, 5, 4, 3, 1\}$
$E = \{3, 5, 6\}$

Then, $A \subset B$; $C \subset B$
$A \subset D$ and $D \subset A$; hence $A = D$
A and C are proper subsets of B
$A \subset A$; $B \subset B$; $C \subset C$; $D \subset D$; $E \subset E$
ϕ is a subset of every set A, B, C, D, E.

But $E \not\subset B$ (E is not a subset of B)

9. Disjoint sets — Two sets which have no members in common are called disjoint sets.

10. Intersection — The intersection (denoted by "$\cap$") of two sets A and B is the set consisting of all the members common to A and B.

Symbolically, $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

If A and B are disjoint sets, then $A \cap B = \phi$.

If two sets are said to intersect (verb form), then their intersection has at least one member; that is, their intersection is non-empty.

Examples: Let $A = \{1, 2, 3, 4, 5\}$
$B = \{3, 4, 6, 7, \}$
$C = \{1, 3, 4, 8\}$
$D = \{10, 11, 12\}$

Then, $B \cap C = \{3, 4\}$
$A \cap C = \{1, 3, 4\}$
$B \cap D = \phi$

11. Union — The union (denoted by "$\cup$") of two sets A and B is the set consisting of all the members which are in at least one of the two given sets A and B.

Symbolically, $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

Observe that the connective "or" in mathematics is used in the inclusive sense; that is, x is a member of A or x is a member of B or x is a member of both A and B.

Examples: Let $A = \{ 1, 2, 3, 4, 5 \}$
$B = \{ 3, 4, 6, 7 \}$
$C = \{ 1, 3, 4, 8 \}$

Then, $A \cup B = \{ 1, 2, 3, 4, 5, 6, 7 \}$
$B \cup C = \{ 1, 3, 4, 6, 7, 8 \}$

12. Cartesian Product

The Cartesian Product of two sets A and B (not necessarily different) is the set of all ordered pairs in which the first coordinate belongs to A and the second coordinate belongs to B.

The Cartesian Product of A and B is denoted as:

$A \times B$ (read "A cross B")

Example: Let $A = \{ a, b, c, \}$ and $B = \{ 1, 2 \}$

Then, $A \times B = \{ (a,1), (b,1), (c,1), (a,2), (b,2), (c,2) \}$

13. Relation

A relation is a set of ordered pairs.

or

A relation from A to B is a subset of $A \times B$.

Example: Let $A = \{ a, b, c \}$; let $B = \{ 1, 2 \}$

Some relations from A to B are:

Relation $R = \{ (a,1), (b,2), (c,1) \}$
Relation $Q = \{ (a,2) \}$
Relation $T = \{ (a,1), (b,1), (c,1) \}$

Of course, $A \times B$ is also a relation.

Also, the empty set, ϕ, is a relation.

14. Domain of a relation

The domain of a relation is the set of all the first coordinates of the ordered pairs of the relation.

15. Range of a relation

The range of a relation is the set of all the second coordinates of the ordered pairs of the relation.

Example: Let Relation $T = \{ (a,2), (b,3), (a,5), (c,2) \}$

Then, Domain of $T = \{ a, b, c \}$
Range of $T = \{ 2, 3, 5 \}$

16. Function : A function from A to B is a special kind of relation such that for each first coordinate there is one and only one second coordinate.

Stated differently, no two ordered pairs of a function may have the same first coordinate. Hence, the first coordinates in the ordered pairs of a function must be different. However, the second coordinate may be the same.

Examples: D = { (1, a), (2, b), (3, c), } is a function.
E = { (1, a), (2, b), (1, c) is not a function; it is a relation.
K = { (1, a), (2, a), (3, a) } is a function--- in a particular, a constant function.

Note: Domain of K = { 1, 2, 3 }
Range of K = { a }

I = { (x, y) | y = x } is a function.

Note: Domain of I is the set of real numbers.
Range of I is the set of real numbers.

17. Tests for a function :

(a) The ordered pair test

A relation is a function provided every first coordinate is different.

(b) The vertical line test

A relation is a function provided every vertical line intersects the graph of the relation in exactly one point. If at least one vertical line intersects the graph in more than one point, then the graph does not represent a function, but a relation.

18. Comparison of f and f(x) : In the function, f = { (x, y) | y = 5x }

(a) the function f is the entire set of ordered pairs.

(b) f(x) is not the function.

(c) "y", "f(x)", "5x" are different names for the second coordinate. Thus the function f might be written in different forms, as:

$$f = \{ (x, y) \mid y = 5x \}$$

or

$$f = \{ (x, f(x)) \mid f(x) = 5x \}$$

or

$$f = \{ (x, 5x) \mid x \text{ is a real number} \}$$

(d) The function is not "y = 5x". Instead, "y = 5x" is a sentence which defines the function. This sentence assigns to each real number used as first coordinate another real number 5 times as large for its corresponding second coordinate.

"f(x)" is read "f at x" or "f of x" or "the value of the function f at x".

The function, $f = \{ (x, y) \mid y = 5x \}$ is read:

the function f defined by $y = 5x$

HOMEWORK ASSIGNMENT

Study assignment: Study pages 1 - 5 in this Review of Elementary Set Concepts

Written assignment:

1. Construct the cartesian product of $A \times B$ where

$$A = \{ r, t, k \} \text{ and } B = \{ 6, 16, 1526 \}$$

2. Construct the cartesian product of $R \times S$ where

$$R = \{ 0, 2, 4, 6 \} \text{ and } S = \{ 1, 3, 5 \}$$

3. Construct the cartesian product of $E \times D$ where

$$D = \{ a \} \text{ and } E = \{ 1 \}$$

Directions for Examples 4 - 13:

Each of the following relations is a relation from R to R where R is the set of real numbers. In each example, complete the following parts:

a. Draw the graph of the relation.
b. State whether the relation is a function.
c. State the domain of the relation.
d. State the range of the relation.

4. $R = \{ (a, y) \mid y = x \}$

5. $S = \{ (x, S(x)) \mid S(x) = \frac{1}{2} x \}$

6. $T = \{ (x, y) \mid y > 5x \}$

7. $U = \{ (x, y) \mid y = x^2 \}$

8. $V = \{ (x, V(x)) \mid V(x) = 2x - 5 \}$

9. $W = \{ (x, y) \mid y = x^3 \}$

10. $X = \{ (x, y) \mid x^2 + y^2 = 25 \}$

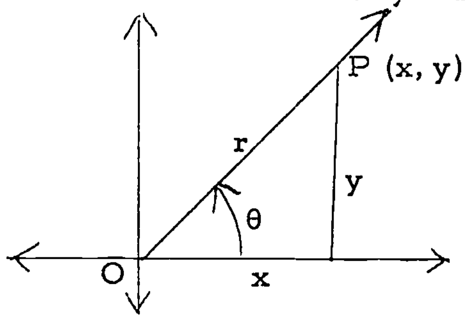
11. $Y = \{ (x, y) \mid xy = 12 \}$

12. $G = \{ (x, G(x)) \mid G(x) = 5 \}$

13. $H = \{ (x, y) \mid x = 3 \}$

Trigonometric Functions

Let P with coordinates (x, y) be a point on the terminal side of an angle with measure θ in standard position. Let $r = \sqrt{x^2 + y^2}$ represent distance of P from the origin.



1. The Sine Function

   Sine (or sin) = $\{ (\theta, \sin\theta) \mid \sin\theta = \frac{y}{r} \}$

   Observe that:

   (1) θ is a number----a number that measures the magnitude of the angle in degrees or radians.

   (2) $\sin\theta$ is a number----the number which is assigned as the second coordinate by the defining sentence

   $$\sin\theta = \frac{y}{r}$$

   (3) $\sin\theta$ is NOT the function.

   $\sin\theta$ is the name of the second coordinate in the sine function.

   $\sin\theta$ is the VALUE of the sine function at θ.

   (4) Each ordered pair of the sine function has real numbers for the first and second coordinates.

   These ideas apply as well to the remaining trigonometric functions.

2. The Cosine Function

   cosine (or cos) = $\{ (\theta, \cos\theta) \mid \cos\theta = \frac{x}{r} \}$

3. The Tangent Function

   tangent (or tan) = $\{ (\theta, \tan\theta) \mid \tan\theta = \frac{y}{x} \}$

4. The Cotangent Function

   cotangent (or cot) = $\{ (\theta, \cot\theta) \mid \cot\theta = \frac{x}{y} \}$

5. The Secant Function

   secant (or sec) = $\{ (\theta, \sec\theta) \mid \sec\theta = \frac{r}{x} \}$

6. The Cosecant Function

   cosecant (or csc) = $\{ (\theta, \csc\theta) \mid \csc\theta = \frac{r}{y} \}$